PROBLEM SECTION

Students are invited to submit solutions to one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who have sent correct solutions by 31/12/64 will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions to problems in the Junior section. All students may submit solutions to problems in the Open section.

JUNIOR

J11. A drill squad contains mn people, arranged in m rows of n people. In each of the n columns so formed, the lightest person is noted, and the heaviest of these is found to be Smith. Then the heaviest person in each row is noted, and the lightest of these is found to be Jones. Of Smith and Jones, who is the heavier?

J12. (i) Show that 120 is a divisor of $n^5 - 5n^3 + 4n$ for every integer n.

(ii) Show that 49 is not a divisor of $n^2 + n + 2$ for any integer n.

J13. ABCD is a convex quadrilateral and P is a variable point. For what position of P is the sum of the lengths $PA + PB + PC + PD$ least?

J14. In the following calculation, consisting of a long multiplication of two 2-digit numbers, followed by the addition of a 3-digit number, all save one of the digits have been obliterated. Find what they must have been.

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             x x
multiply     x x
           x x x
        x x x
        x x x x
add         1 x x
      x x x x x
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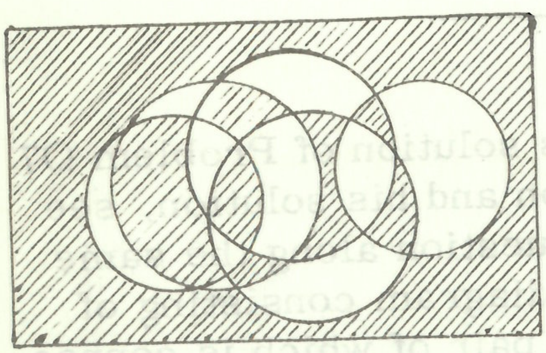
OPEN SECTION

O15. Find all solutions of the following simultaneous equations:

$$xy + wz = 20$$
$$xz + wy = 20$$
$$xw + yz = 20$$
$$x + y + z + w = 14.$$

O16. The accompanying diagram shows a map obtained by drawing five complete circles.



The resulting regions into which the plane has been divided are coloured with two colours in such a way that regions with a positive lenght of boundary in common have different colours. Show that two colours are sufficient for every map obtained in this way, irrespective of the number of circles it contains.

O17. P is a simple polyhedron with more than eight faces. Each face has n edges, and r edges meet at each vertex. Show that P has exactly 30 edges. (Hint: First use Euler's Formula (p 5) to obtain $1/n + 1/r = 1/2 + 1/E$).

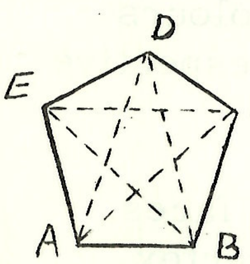
O18. $P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ is a polynomial with integral coefficients $a_1, a_2, \ldots, a_n$. For some integer b, P(b) and P(b + 1) are both odd numbers. Prove that the equation P(x) = 0 has no integral solutions.

O19. In chess, is it possible for the knight to start at the lower right hand corner of the board and finish at the upper left hand corner, lighting exactly once on every square on the board?

O20. Four brothers A, B, C, and D, sold a herd of cattle, each beast fetching as many pounds as there were animals in the herd. They shared the money in the following way. First A took £10, then B took £10, then C, then D, then A again, and so on. Eventually, it being D's turn, there was less than £10 remaining. However, A, B, and C each returned a sum of money to D so that the division was perfectly fair. How much did each return?

Owerchuk's Problem.

G. Owerchuk, in submitting his solution of Problem O7 (Parabola, Vol1, No 1; for the question and his solution, see p. 18 of this issue), raised a further question along the same lines. He produced the accompanying diagram consisting of five points, every pair of which is connected by a straight line segment, either dotted or unbroken. There is no triangle with vertices selected from A, B, C, D, and E whose sides are all dotted lines, or all unbroken lines. Problem O7 shows that such a diagram with six points is impossible; i.e. five is the largest number of points which may be completely connected by lines of two colours in such a way as to avoid one-colour



triangles.

Owerchuk now asks:- What is the largest number n_r of points (no three of which are collinear) which may be completely connected with straight line segments using r different colours, in such a way that no one-colour triangles result? The above example shows that $n_2 = 5$, and it is even easier to see that $n_1 = 2$.

It does not appear to be easy to answer this question completely. A partial answer is contained in the following statements:-

(i) $n_r \leqslant r.n_{r-1} + 1$

(ii) hence $n_r \leqslant r!\,(1 + 1/1! + 1/2! + \ldots + 1/r!)$

(The symbol k! means 1. 2. 3. . . . k)

(iii) from (i) or (ii), putting $r = 3$, we obtain $n_3 \leqslant 16$.

It is possible to construct a diagram of sixteen points completely connected by lines of three colours which contains no one-colour triangles, and this shows that $n_3 = 16$.

Thus the formula on the right hand side of (ii) gives the exact value of n_r for r equal to one, two, or three. For r equal to four, (ii) gives $n_r \leqslant 65$, but I do not know if this is again the exact value.

We invite readers to prove the statements (i), (ii), and (iii), and if possible find out more information concerning the question. In particular, it would be useful to have an inequality like (ii) involving a reasonably good approximation to n_r, but in the opposite direction. We would like to be able to publish the researches of some student(s) about this problem in next issue of Parabola.

How to be a Space Traveller at Little or No Expense.

Several countries to-day are spending vast sums of money to try to discover ways of getting into outer space. Is all this expense really necessary? Try this simple method in your backyard next week-end and put Australia to the front in the Space Race.

Beg a large sheet of paper - say about one two-hundredth of an inch thick - and place it on the ground.

Stand on it.

Fold it double and stand on it again. You are now twice as high as you were before.

Do this doubling business fifty times, and don't forget to stand on the folded paper every time.

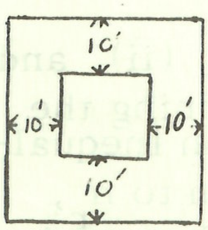
You are now about eighty million miles away from the Earth - more than three hundred times as far away as the moon.

Don't forget to ask Mum for a packed lunch before you start on the job.

If you don't believe it, work it out for yourself.

(Contributed by Prof. G. Bosson)

PREPARED



How was a boy scout equipped only with two planks each nine feet long, able to cross the square moat shown in the diagram?

(Answer p.28)