PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class, and school. Model solutions and the names of those who send correct solutions by 31/8/65 will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions or problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J31. Prove that, given five consecutive positive integers, it is always possible to find one which is relatively prime to all the rest.

J32. A regular polygon is cut from a sheet of cardboard. A pin is put through the centre to serve as an axis about which the polygon can revolve. Find the least number of sides that the polygon can have in order that revolution through an angle of $47\frac{1}{2}$ degrees will put it in co-incidence with its original position.

J. 33. (i) Prove that if a and b are any two positive numbers

$$ab \leqslant \left(\frac{a+b}{2}\right)^2 \leqslant \frac{a^2+b^2}{2}.$$

When do the equal signs hold.

(ii) Prove that if $a + b = 1$, where $a > 0$, $b > 0$, then

$$\left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2 \geqslant \frac{25}{2}.$$

OPEN SECTION

O34. Three people, A, B and C, entered a competition. After the event, A reported "B was second, C was first". B said, "A was second, C was third". C said, "A was first, B was third". Each person's report contained one true statement and one false one. Which of A and B performed better in the competition?

(The next three problems can be attempted only after the article "Almost all Real Numbers are Transcendental" has been read.)

O35. (i) A and B are disjoint sets (i.e., sets with no elements in common), each of which has cardinal number $\aleph_0$. Show that the set C which is the union of A and B (i.e., the set which contains all the elements in A and all the elements in B, and no other elements) also has cardinal number $\aleph_0$.

(ii) If $A_1, A_2, A_3, \ldots, A_n \ldots$ are disjoint sets, each of which has cardinal number $\aleph_0$, prove that the union of all the sets A_n is a set whose cardinal number is $\aleph_0$

O36. If A is a set whose cardinal number is c, and B is a subset of A whose cardinal number is $\aleph_0$, show that the set (A-B), consisting of the elements of A which are not contained in the subset B, has cardinal number c.

Deduce that the set of transcendental numbers has cardinal c.

O37. Show how to set up (1-1) correspondences between the points (o, y), $o < y < 1$, on the y axis and

(i) the positive half of the x-axis

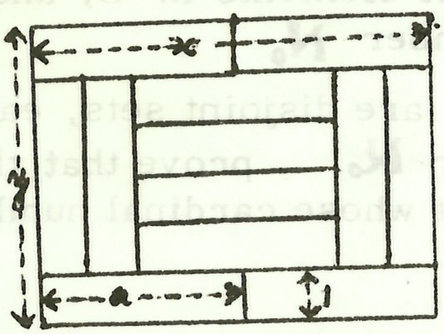
(ii) the entire x-axis

(iii) all points (x, y) in the unit square $o < x < 1$, $o < y < 1$.

O38. Prove that if all coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers, then the roots of the equation cannot be rational.

O39. In a certain community, any person who is married and plays tennis does not own a car, but if a man plays tennis, he owns a car. Any person who does not own a car is either single and a tennis player or female and not a tennis player. All the car owners who are still single are female, and in fact, there is no single female tennis player who does not own a car. What can one conclude about a person in the community who is known to play tennis?

O40.



The diagram shows a rectangular floor x units long and y units wide, which is covered with tiles $1 \times a$ units. (a, x, and y are all positive integers.) Prove that such a covering is possible (without cutting any tiles) if and only if "a" divides either x or y exactly.

(The following puzzle "Dominoes" may suggest a possible method of tackling this difficult problem.)

Dominoes.

A set of dominoes are of such a size that each domino will exactly cover two squares of a certain chessboard. Is it possible to place 31 dominoes on the board so that the only squares not covered are a pair of diagonally opposite corner squares?

Answer p. 13