not occur on Friday, because on Thursday, since the privates know that the parade could not occur on Saturday, it would have to occur on Friday. And so, by induction, they convince themselves that no A-parade can occur at all, and are very surprised when one does indeed occur on the Tuesday! Where did they err?

M. Arbib.

PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by 31/12/65 will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR.

J31 As inquiries revealed that several students did not know the meaning of the phrase "relatively prime", we repeat the question.

Prove that, given five consecutive integers, it is always possible to find one which is relatively prime to all the rest.

Two integers are relatively prime (or, one is prime to the other) if they have no common factor greater than one.

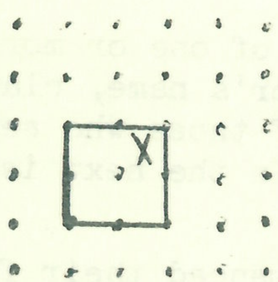
(S. Byrnes has already submitted an excellent solution of this problem).

J41 In the following equation:-

$$29 + 38 + 10 + 4 + 5 + 6 + 7 = 99$$

the left hand side contains every digit exactly once. Either find a similar expression (involving only + signs) whose sum is 100, or prove that it is impossible to do so.

J42 (i) X is a point at the centre of a square array of dots with 2n dots in each side. (In the diagram shown, $n = 3$). The diagram shows a square whose sides lie in the rows or columns of dots, and which encloses the point X. How many such squares may be drawn if $n = 1, 2, 3, \ldots, n$?



(ii) How many rectangles enclosing X may be drawn through the dots?

J43 We call $1 - x$ the complement of x, and, if $x \neq 0$, we call $1/x$ the reciprocal of x.

Starting with the number $\frac{1}{3}$, we form its reciprocal, 3, and its complement, $\frac{2}{3}$, and we find as many numbers as possible by repeatedly taking reciprocals or complements of numbers we have found so far. Which numbers are obtained by this process? Will the number of numbers obtained be different if we start with a different number (other than 0 or 1)?

OPEN SECTION

O44 Prove that if the sum of the fractions $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2}$ (where n is a positive integer) is put into decimal form in the scale of ten, the resulting decimal does not terminate.

045 (i) The period (recurring block of digits) of a recurring decimal is of length n and begins at the $k + 1^{st}$ place to the right of the decimal point. Let

$$\frac{A}{10^k}$$

be the part preceding the period, and let P, an integer of n digits, be the period (e.g., in $3.6\overline{87}$, $k = 1$, $n = 2$, $A = 36$, $P = 87$). Prove that

$$A(10^n - 1) + P$$

is not divisible by 10.

(ii) Let $\frac{m}{2^\alpha 5^\beta q}$ $(\alpha \geq 0, \beta \geq 0)$

be a fraction such that q is not divisible by 5 or 2, and let k be the greater of the two integers α, β. Using (i), prove that in the decimal expansion of this fraction the period begins at the $k + 1^{st}$ place to the right of the decimal point.

046 Carry out the division processes for finding the decimal expansions of

$$\frac{a}{41} \quad \text{for} \quad a = 1, 2, \ldots, 40,$$

and list the distinct remainders in each case. The table you will obtain begins as follows:

a	remainders
1	1, 10, 16, 18, 37;
2	2, 20, 32, 33, 36;
3	3, 7, 13, 29, 30;
	....

After observing the completed table, prove that, for p prime,

(i) in the division processes for $\frac{a}{p}$ for $a = 1, 2, \ldots, p-1$, each set of remainders occurs for exactly n values of a, where n is the length of the period of $\frac{a}{p}$;

046 cont'd:

(ii) $p - 1$ is divisible by n ;

(iii) 10^{p-1} is divisible by p. (This is a particular case of Fermat's theorem.) (Remainder table need not be sent in).

047 Prove that the Fibonacci sequence,

$u_1 = 1$, $u_2 = 1$, $u_3 = 2$, $u_4 = 3$, $u_5 = 5$, $u_6 = 8$, $u_7 = 13$

..... , where $u_{n+1} = u_n + u_{n-1}$ for $n \geq 2$,

has the following properties.

(i) $u_n^2 - u_{n-1} u_{n+1} = (-1)^{n-1}$

(ii) u_n is even if and only if n is a multiple of 3.

(iii) $u_n = u_{r+1} u_{n-r} + u_r u_{n-r-1}$, $1 \leq r \leq n-2$

(iv) u_k is a factor of u_n if and only if k is a factor of n .

048 For the Fibonacci sequence, if p is any prime number, there is a value of $n \leq p + 1$ such that p is a factor of u_n .

049 Construct a Hadamard configuration for $m = 3$, $n = 11$. (See the article "Unsolved Problems 2 - The Hadamard Problem).

050 Solve the cross number puzzle on page 7.