# UNSOLVED PROBLEMS 3.

From the earliest times of its development, geometry has offered a number of difficult and quite puzzling problems which remained unanswered until comparatively recent times. The Greek geometers have bequeathed to posterity four famous problems which they were unable to solve in spite of their highly developed geometrical skill.

1. Squaring of the circle. Given the radius of a circle, construct a distance equal in length to the perimeter. "Construction" here as in the following problems means: by means of the conventional operations performed with ruler and compasses.

2. Trisection of the angle. Given an arbitrary angle $\alpha$, construct $\frac{1}{3}\alpha$, that is an angle $\beta$ such that $3\beta = \alpha$. There are of course special angles such as the right angle which can be trisected by construction, but the problem is to find a general method of construction which will trisect any angle.

3. Duplication of the cube. Given the side of a cube, construct the side of another cube whose volume is twice the volume of the first one.

4. The problem of parallels. Given a line and a point outside the line, prove that there is exactly one line through this point which is parallel to (i.e., has no point in common with) the given line. This was stated as a postulate by Euclid, but it had the form of a "theorem" rather than a "self-evident truth", and people wondered whether it could perhaps be proved from the other, more obvious looking postulates.

It took mathematics more than 2000 years to give an answer to these questions. With the advent of analytic geometry in the seventeenth century all problems of Euclidean geometry could be formulated in arithmetical terms and their solution became a matter of algebraic manipulations. In the case of the four classical problems, however, the required manipulations led to some quite deep and subtle algebraic problems which were not cleared up completely until the middle of the last century.

Surprisingly the answer turned out to be negative in all four cases: the first three constructions cannot be carried out by the use of ruler and compasses alone, and the parallel postulate cannot be deduced from the other Euclidean axioms.

As I said earlier, in principle all problems of geometry can be arithmetised and solved by algebraic manipulations, but often the algebra involved is so cumbersome that it is of little help, and even to-day there exist unsolved problems in Euclidean geometry. I mention here one which was solved only quite recently. The problem is this:

> A triangle ABC is divided into four smaller ones by means of the points X, Y, Z on the sides AB, BC, CD respectively. Prove that the triangle in the middle, XYZ, cannot have the smallest circumference.
> (There is one exception: when X, Y, Z are the midpoints of the respective sides then all four triangles have the same circumference).

If "circumference" is replaced by "area", the problem becomes much easier. (See problem 059).

In the next issue I shall discuss some other unsolved geometrical problems.

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<u>PROBLEM 059.</u>

The triangle ABC is divided into four smaller triangles by means of the points X, Y, Z on the sides AB, BC, CD respectively. Prove that the middle triangle XYZ cannot have the smallest area (though it can be equal in area to the smallest corner triangle in certain cases).