PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by 16th May, 1966, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J51 Prove that a number which consists of 3^n equal digits is divisible by 3^n. [e.g. if n=1, 555 is divisible by 3; if n=2, 888, 888, 888 is divisible by 9, and so on].

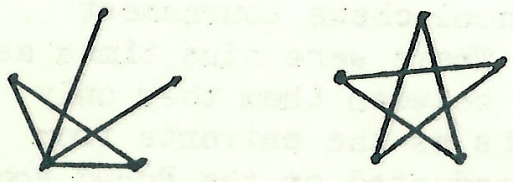
J52 Prove that if the number x is given by 0.999 ..., where there are at least 50 nines, then $\sqrt{x}$ also has 50 nines at the beginning.

J53 If $x + y + z = 0$ simplify the following expression:

$$\left(\frac{y-z}{x} + \frac{z-x}{y} + \frac{x-y}{z}\right)\left(\frac{x}{y-z} + \frac{y}{z-x} + \frac{z}{x-y}\right).$$

OPEN

O54



These diagrams show two ways of drawing 5 line segments connecting the vertices of a convex pentagon with the property that every pair of lines intersect. It is impossible to draw 6 line segments (each an unproduced side or diagonal of the pentagon) with this property. Prove that in fact it is not possible to draw (n + 1) sides or diagonals of a convex polygon with n vertices in such a way that every pair intersect.

055 Show that the binomial coefficients, ${}^{n}C_{r}$, defined by

$$(a+b)^n = a^n + {}^{n}C_1\, a^{n-1}b + {}^{n}C_2\, a^{n-2}b^2 + .. + {}^{n}C_r\, a^{n-r}b^r + .. + {}^{n}C_n\, b^n ,$$

satisfy the following identities for any positive integer n .

(i) $${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \ldots + {}^{n}C_n = 2^n .$$

(ii) $${}^{n}C_0 - {}^{n}C_1 + {}^{n}C_2 - \ldots = 0 .$$

(iii) $$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \ldots + ({}^{2n}C_{2n})^2 = (-1)^n\, {}^{2n}C_n .$$

(iv) $$({}^{2n+1}C_0)^2 - ({}^{2n+1}C_1)^2 + ({}^{2n+1}C_2)^2 .. - ({}^{2n+1}C_{2n+1})^2 = 0 .$$

(v) $$({}^{n}C_0)^2 + ({}^{n}C_1)^2 + ({}^{n}C_2)^2 + \ldots + ({}^{n}C_n)^2 = {}^{2n}C_n .$$

056 Duplicate, and generally n-plicate the square. That is, given a square in the plane, construct one with an area n times the original square.

057 A cylindrical hole is drilled through the centre of a sphere. If the hole is 6" long find the volume of the remaining portion of the sphere.

058 Answer questions 4 and 8 at the end of the article on Braids.

059 See page 16.

060 All entrants for a certain school chess tournament were from form IV or form V. There were nine times as many from the lower form, but between them they only scored 4 times as many points as the entrants from form V. The tournament was conducted on the Round Robin system (i.e., each entrant played all the others once, scoring 1 for a win, $\frac{1}{2}$ for a draw and 0 for a loss). What was the winner's score?