PROBLEM SECTION.

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by August 1966, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J61 A strip of pasture land is divided into three paddocks whose areas are 35 acres, 10 acres and 65 acres.

On a certain date the owner places 17, 7 and 23 head of cattle (respectively) in the paddocks. The feed in the first is exhausted after 7 weeks, and that in the second after 4 weeks. For how long can the 23 cattle be grazed in the third paddock?

[You will need to assume that initially there is the same amount of food/unit area and that new feed grows at the same constant rate in all three paddocks.]

J62 A launch which keeps up a steady speed through the water leaves the boat house and travels upstream. After 2 miles it meets some driftwood floating down stream. The launch travels upstream for another hour and then returns to the boat house, arriving at the same moment as the driftwood. Calculate the speed of the stream.

J63 Prove that the square of every prime number greater than 3 leaves a remainder of 1 when divided by 12.

OPEN

064 In any party of n people we can always find two who have shaken hands with the same number of other people in the party.

065 See page 3.

066 If $a_1, a_2, \ldots\ldots, a_n$ are positive numbers we define the arithmetic mean $A = \dfrac{a_1 + a_2 + \ldots\ldots + a_n}{n}$; the geometric mean $G = \sqrt[n]{(a_1 a_2 \ldots a_n)}$; the harmonic mean $H = \dfrac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots\ldots + \frac{1}{a_n}\right)}$, and the root mean square $R = \sqrt{\frac{1}{n}\left(a_1^2 + a_2^2 + \ldots\ldots + a_n^2\right)}$. A theorem asserts that $A \geq G$ regardless of the values of $a_1, \ldots\ldots, a_n$. Assuming this, determine the order of all four quantities, and prove that it is independent of the values of $a_., a_n, \ldots a_n$.

067 Given n positive numbers $x_1, x_2, \ldots\ldots, x_n$ prove that

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_4} + \ldots\ldots + \frac{x_{n-1}}{x_n} + \frac{x_n}{x_1} \geq n$$

068 Given a square of side 1 unit of length, and a point P on one side of the square, show how to construct, with ruler and compasses only, an equilateral triangle inscribed in the square with one vertex at P. Also find the smallest and largest possible lengths of the side of an equilateral triangle inscribed in the square.

069 A ten pin bowler has eleven weeks to prepare for a tournament. He decides to train by playing at least one game every day, but he cannot afford to play more than twelve games in any one week. Prove that in some number of consecutive days he plays exactly 20 games.

070 A lives in a street in which there are houses numbered from 1 to 30. B wants to know the number of A's house.

B asks A: "Is it greater or less than 15?" A answers, but he lies.

B asks A: "Is it a perfect square?" A answers truthfully.

B says to A: "Is it odd or even? That will fix it definitely."

"Even", says A truthfully. B tells him the number, but he is wrong. What was the number of A's house?

Painted Faces. A wooden cube of side 3" is painted red. It is then sawn up into cubes of side 1". How many of these small cubes have three red faces? Two red faces? One? None? Also give the four answers if the original cube has a side of length n inches.

(Answer p.32)