Parabola Volume 3, Issue 3 (1966)

PROBLEM SECTION.

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by December 31st, 1966, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J71 Find a four digit number which on division by 149 leaves a remainder of 17 and on division by 148 leaves a remainder of 29.

J72 Find the sum of the coefficients of the polynomial obtained after expanding and collecting the terms of the product $(1-5x+5x^3)^{100}(1-2x+x^4)^{10}$.

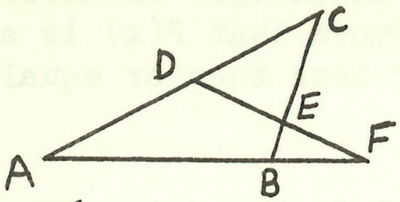
OPEN

O73 Prove that if the positive integers a_1, a_2 and a_2-a_1 are all relatively prime to n, the cyclic Latin squares on n symbols constructed from the first row by moving the symbols a_1 (or a_2) squares to the left to obtain each succeeding row, are orthogonal.

O74 In the array shown (which, if the plus signs are omitted is exactly the Graeco-Latin square of Fig.2A, p. 2) putting $a = 1$, $b = 3$, $c = 2$, $\alpha = 3$, $\beta = 0$, $\gamma = 6$ gives the familiar magic square using the first nine positive integers whose rows, columns and diagonals all sum to 15. Construct magic squares using the first 16 positive integers, and the first 25 positive integers, with the help of Graeco-Latin squares of order 4 and 5 respectively. (This problem explains why the title of Euler's article mentioned magic squares.)

$a+\alpha$	$b+\beta$	$c+\gamma$
$b+\gamma$	$c+\alpha$	$a+\beta$
$c+\beta$	$a+\gamma$	$b+\alpha$

075 In the figure shown, it is known that AB* + BE* = AD* + DE*. Prove that AC* + CE* = AF* + FE*. [Here AB* stands for the length of the interval AB etc.]



076 A man lost in the desert hears a train whistle due west of him. Although the track is too far away to be seen he knows that it is straight, and runs in a direction somewhere between south and east, (but does not know its exact course). He realises that his only chance to avoid perishing from thirst is to reach the track before the train has passed. In which direction should he travel to give himself the greatest possible chance of survival? (You are expected to assume that the man and the train both move at constant speeds, the former more slowly than the latter).

077 If

$$\begin{aligned} K = {} & (x_1 - x_2)^2 + (x_1 - x_3)^2 + \ldots + (x_1 - x_{2m})^2 \\ & + (x_2 - x_3)^2 + (x_2 - x_4)^2 + \ldots + (x_2 - x_{2m})^2 \\ & + (x_3 - x_4)^2 + \ldots + (x_3 - x_{2m})^2 \\ & + \ldots\ldots\ldots \\ & + (x_{2m-1} - x_{2m})^2, \end{aligned}$$

$$= \sum_{\substack{i,j=1 \\ i<j}}^{2m} (x_i - x_j)^2$$

find the maximum possible value of K if it is known that each x_i $(i = 1,2,\ldots 2m)$ has the value 0 or 1.

078 $P(x)$ is a polynomial whose value is an integer whenever x is an integer greater than 100. Prove that $P(x)$ is also integer valued when x is any integer less than or equal to 100.

079 Prove that if the polynomial

$$a_0x^7 + a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 + a_5x^2 + a_6x + a_7$$

whose coefficients are all integers, has for seven different integral values of x the value $+1$ or -1, then it cannot be factorised as the product of two polynomials with integral coefficients.

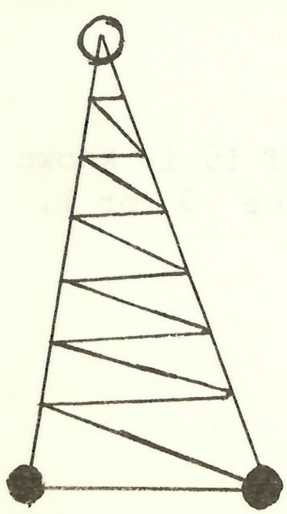
080 You are provided with a pan-balance (but no weights) and n objects. Let W_n be the minimum number of weighings of one object against another which enables you to be certain of arranging the objects in increasing order of weight. [Hence $W_2 = 1$, $W_3 = 3$, and $W_4 = 5$]. Show that $W_5 = 7$.

A general formula for W_n is unknown. However, obtain the following upper bound for W_n:-

if $2^{e-1} < n \leqslant 2^e$ then $W_n \leqslant en + 1 - 2^e$

....

Hare and Hounds



Here is an easy game for two players. One player moves the hare (the white button), his opponent moves either one of the "hounds", (the black buttons). The players move alternately, a move consisting in shifting a button along a line to the next intersection The hounds win if they corner the hare at his starting point. The hare wins if he escapes to the bottom of the diagram. With correct play the hounds ought to succeed.