PROBLEM SECTION.

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by April 30th, 1967, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J81 The lengths of the sides of a right angled triangle are natural numbers. The length of one side is 14. Find the lengths of the other sides.

J82 Solve the equation $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$.

OPEN

O83 Show that $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ are never three terms in the same arithmetical progression.

O84 Show how to construct, using straight edge and compasses only, an equilateral triangle whose vertices lie on three given parallel straight lines.

O85 A man runs up a certain down-moving escalator and reaches the top after taking 125 steps. He then walks down the escalator slowly, taking one step in the time it took to take 5 steps on the previous trip. He reaches the bottom after 50 steps. How many steps would be visible if the escalator stopped running?

086 Three men A, B and C decide to fight a pistol duel along the following lines. They will first draw lots to determine who fires first, second and third. After positioning themselves at the vertices of an equilateral triangle, they will fire single shots in turn and continue in the same cyclic order until two of them have been hit. The man whose turn it is to fire may aim wherever he pleases. Once a man has been hit, whether killed or not, he takes no further part in the duel. All three men know that A always hits what he aims at, B is 80% accurate, and C is 50% accurate.

Assuming that all three adopt the best strategy, and that no one is hit by a shot not aimed at him, who has the best chance to escape unscathed? What is the exact probability of escape of each of the three men?

087 Three prisoners A, B and C occupy condemned cells in a certain gaol. A questions the warden concerning a rumour that one of them is to be pardoned. The warden confirms the rumour but says that he has not been given authority to announce which prisoner is to live. A, finding the warden adamant, at last prevails upon the warden to name one of the other two prisoners who is going to be executed. "If they are both going to die", he says, "say whichever of B and C you please". The warden sees nothing wrong with the suggestion and tells A that B has not been pardoned. A now claims that his probability of survival has increased from $\frac{1}{3}$ to $\frac{1}{2}$. He manages to slide a note with the information under the wall into the next cell, occupied by C, who also congratulates himself that his survival chances have now increased from $\frac{1}{3}$ to $\frac{1}{2}$.

Is A's reasoning correct? If not, what are the survival chances of each of the three prisoners?

088 Answer problems 8 and 9 at the end of the article on continued fractions.

089 Prove that if a plane figure has exactly two axes of symmetry, they are at right angles.

090 A tropical island is inhabited by 5 men and a pet monkey. The men collect a large pile of coconuts, intending to distribute them equally amongst themselves on the following morning. During the night, however, one man, A, quietly goes to the coconuts and divides them into 5 equal piles. There is one coconut left over which he gives to the monkey. He then puts four of the piles back together and hides the fifth pile before going back to bed. Later a second man, B, does exactly the same thing and so do C, D and E; i.e. each divides the heap left by the previous visitor into 5 equal piles, finds one coconut left over which he gives to the monkey, hides one pile and heaps together the remaining four. In the morning, the residual heap of coconuts is divided equally amongst the five men, one coconut left over being again given to the monkey.

What is the smallest number of coconuts which could have been contained in the original heap?

A chessboard problem

A chessboard is 16 inches square. A circle is drawn on the board with black ink but remains invisible because its circumference lies entirely on the black squares. What is the largest possible radius of such an invisible circle?

(Answer page 24.).