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PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by August 31st, 1967, will be published in the next issue of PARABOLA.

Only those sutdents who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J91 (i) Find all whole numbers such that when the third digit is deleted, the resulting number divides the original one.

(ii) Find all whole numbers with initial digit 6 such that when this initial digit is deleted, the resulting number is 1/25 of the original number.

J92 Solve the equation

$$|x+1| - |x| + 3|x-1| - 2|x-2| = x+2$$

[The symbol $|a|$ is called the absolute value of a and is defined by $|a| = a$ if a is positive, $|a| = -a$ if a is negative, so that $|a|$ itself is never negative].

OPEN

O93 Show how to dissect a regular hexagon into 5 pieces which can be rearranged to make a square.

O94 Show that every triangle can be dissected into 7 acute angled triangles.

095 Prove that the four pieces of Dudeney's dissection of the equilateral triangle do fit together exactly to make a square.

096 Show that $\sum_{k=1}^{n} \frac{1}{k}$, i.e. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n}$

is never exactly equal to a whole number for any integer n greater than 1.

097 A game is played with two people and a heap of matches. There are initially an odd number of matches in the heap. The players move alternately, each move consisting in the removal of 1, 2, or 3 matches from the heap. The winner is the player who ends up with an odd number of matches when there are no matches remaining in the heap. If there are initially 61 matches in the heap which player ought to win with correct play, the first to move or the second? Describe his winning strategy.

098 Another game is played with two people as follows. The first player, A, writes down on separate pieces of paper any number of distinct integers. He then turns these face down on a table. The second player, B, does not know which numbers have been written down. He begins turning the pieces of paper over one at a time. At any stage he can stop and say "This number which I have just turned face up, is the largest number you wrote down". If he is right he wins the game, otherwise he loses. What is B's best strategy, if A has used 20 pieces of paper, or n pieces of paper? What is his probability of winning?

099 A number of towns $P_1, P_2, \ldots P_n$, no three collinear, are connected by a system of roads with the following properties. Given any pair of towns there is a straight road joining the two towns. There are no road junctions except at the towns. (Roads may cross each other at other points, but fly overs are used which make it impossible to leave one road and enter the other). All the roads are one way but the directions of traffic in the roads are so chosen that it is possible to reach any town from any other. Prove that it is possible to leave P_1 and drive in a circuit so that on returning to P_1, every other town has been visited exactly once.

0100 Three feet four inches above the centre of an end wall of a rectangular room there is a smear of honey. (I don't know how it got there). A very mathematically minded ant emerges from a hole in the opposite end wall 3'4" below the centre. He calculates that to reach the honey there are eight different paths all 27'1" in length, and that every other route is longer.

What are the dimensions of the room?

(The idea behind this puzzle goes back again to H.E. Dudeney, but blame for an extra twist belongs to M. Kraitchik.)