PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by October 31st, 1968, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J121 The numbers a,b,c,d and e are consecutive integers, each smaller than 10,000. Determine these numbers if

$a + b + c + d + e = x$ is a perfect cube and
$b + c + d = y$ is a perfect square.

J122 A square ABCD has sides of length a and diagonals of length d. A triangle APQ is drawn such that P is on the side BC, Q is on the side CD and AP = AQ. Let this triangle have perimeter p. Prove that

a) p is greater than 2d
b) p is less than 4a
c) p is less than or equal to 2a + d.

J123 n! (read n factorial) is defined to be the product of all of the integers from 1 to n inclusive i.e. $n! = 1\times2\times3\times4\times\ldots\times(n-1)\times n$.

a) which is the larger, $\sqrt[8]{8!}$ or $\sqrt[9]{9!}$?
b) simplify the sum $1(1!)+2(2!)+3(3!)+\ldots+n(n!)$.

OPEN

0124 Find the shortest distance from the point $A(0,2)$ to that part of the parabola $y = x^2$ lying between $x = 0$ and $x = 1$.
You must prove that the distance found is the shortest.

0125 Simplify $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$.

0126 A container with the rectangular base ABCD has a transparent semi-cylindrical cover. A thin elastic thread joining the diametrically opposite vertices A and C is stretched over the cover - it may be assumed that the thread takes up a position such that its length is minimized. Vertical light rays cast a shadow of the thread upon the base. Determine the shape of the shadow.

0127 Three motorists A, B and C often travel on a certain highway, and each motorist always travels at a constant speed. A is the fastest of the three and C the slowest.
One day when the three travel in the same direction, B overtakes C, five minutes later A overtakes C and in another three minutes A overtakes B. On another occasion when they again travel in the same direction, A overtakes B first, then, nine minutes later, overtakes C. When will B overtake C?

0128 A pirate decided to bury treasure on an island near the shore of which were two similar boulders A and B and, further inland, three coconut trees C_1, C_2, C_3. Stationing himself at C_1, the pirate laid off C_1A_1 perpendicular and equal to C_1A and directed outwardly from the perimeter of triangle AC_1B.

Cont.

He similarly laid off C_1B_1 perpendicular and equal to C_1B and also directed outwardly from the perimeter of triangle AC_1B. He then located P_1, the intersection of AB_1 and A_1B. Stationing himself at C_2 and C_3, he similarly located points P_2 and P_3, and finally buried his treasure at the circumcentre of triangle $P_1P_2P_3$. Returning to the island some years later, the pirate found that a storm had obliterated all the coconut trees on the island. How might he find his buried treasure?

0129 A resident of Las Vegas, on his deathbed, held the following conversation with a friend who liked to solve problems.
"Over the past few years, I have saved every silver dollar that came into my hands. When I reached one hundred, I tied them in a bag. The first three hundred accumulated very fast and I hoped to reach a thousand, but I didn't make it. The several bags, each containing one hundred dollars, are in the closet of the next room. What I want you to do is to visit my son, who is a minor, on his next and each succeeding birthday and give him the number of dollars which equals his age. I figure that on your last trip you will have just used up all of the money."
"That's interesting," said his friend. "Before I see the bags, let me try to figure out how many trips will be required." After an interval, the friend said, "L'll have to know your son's age."
He was told. Then he said, "I now know how many trips will be required."
How many will he have to make?

0130 "Did your teacher give you that problem?" I asked.
"It looks rather tedious."
"No," said Tim, "I made it up. It's a polynomial with my age as a root. That is, x stands for my age last birthday."
"Well, then," I remarked, "it shouldn't be so hard to work out - integer coefficients, an integer root. Suppose I try x = 7 ... No, that gives 77."
"Do I only look 7 years old?" demanded Tim.
"Well, let me try a larger integer ... No, that gives 85, not zero."
"Oh, stop kidding!" said Tim, looking over my shoulder. "You know I'm older than that!"
How old is Tim?

Puzzle: Find the Digits

Each of the letters A,B,C,D,E,F,G,H stands for a digit and no two letters stand for the same digit. The following represents the multiplication of the number ABC by the number DE:

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   A B C
     D E
  ------
   F E C
 D E C
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 H G B C
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Find the digits represented by the letters.

(Answer p. 33)

The School Mathematics Competition

This year's was the seventh such competition to be held. It attracted a record number of entrants, 804 in the junior division and 570 in the senior division, coming from more than 200 schools throughout the state.

In the junior section, first prize went to N. C. Wormald (Sydney Technical H. S.), who answered every question correctly. I think this is the first time a perfect score has been achieved in the junior division, whereas three senior entrants in earlier competitions have matched the feat. Second prize was awarded to G. J. Williams(Newcastle B. H. S.), and third prize to Alison C. Stewart (Cheltenham G. H. S.). Others who succeeded with about three of the five questions were C. J. Bennett (Syd. Grammar), I. A. Pollard (S. C. E. G. S.), S. A. Edwards (Syd. Grammar), and D. G. Marr (Newcastle B. H. S.)

In the senior division, first prize was shared by P. J. Bardsley (Lyneham H. S.) and J. S. Richardson (Asquith B. H. S.), each with three questions correct. Third prize went to J. E. Dawson (Syd. Grammar).

Our congratulations to all these very gifted mathematicians, and to all the others (in some cases, no doubt, equally gifted but unluckier on the day) who earned prizes or certificates. Solutions of all the competition problems are presented in this issue of Parabola. Incidentally, the new problems set in this issue have been taken from similar competitions held this year in South Australia and Tasmania.

The Frustrated Hiker

A man walks 10 miles south, then 10 miles west, and then 10 miles north. He is then back at his starting point. From where must he have started?

(Answer p. 33)