

PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by Aug. 31st, 1969, will be published in the next issue of PARABOLA.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J131 (i) How many lines equidistant from three given points can be drawn in the plane?

(ii) Four non-coplanar points are given. How many planes equidistant from all four points can be found?

J132 Paints of six different colours are available. In how many different ways is it possible to paint a cube so that all faces have different colours. (Two colourings are considered the same when one can be obtained from the other by rotating the cube).

OPEN

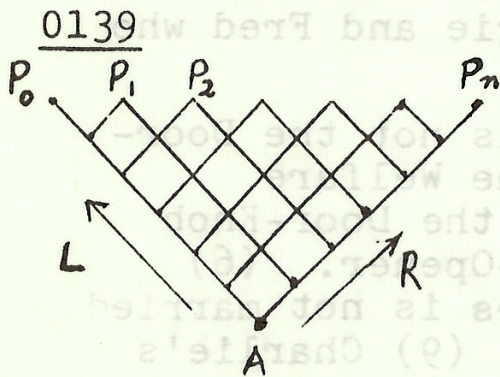
0133 - 4 Solve the problems at the end of the article concerning isosceles triangles (p. 7).

0135 Four points in the plane are given, not all on the same straight line, and not all on a circle. How many straight lines and circles can be drawn which are equidistant from these points?

0136 In how many ways can postage of $4m$ cents be paid using 1 cent, 2 cent and/or 4 cent stamps? What about postage of $4m + 1$ cents, $4m + 2$ cents and $4m + 3$ cents?

0137 There are n people at a party, and n handshakes are made (no pair of people shaking hands twice). Prove that, for some r , $3 \leq r \leq n$, it is possible to find r people P_1, P_2, \dots, P_r such that P_1 has shaken hands with P_2 , P_2 with P_3, \dots, P_{r-1} with P_r and P_r with P_1 .

0138 There are $2n$ people at a party, and $n^2 + 1$ handshakes are made (no pair of people shaking hands twice). Prove that it is possible to find 3 people, each of whom has shaken hands with the other two.



A network of roads is shown in the diagram. A group of 2^n people leave A, half going in the direction L and half in the direction R. At the first intersection each group splits up, half going in direction L and half in direction R. The same behaviour is repeated at each intersection. Let P_0, P_1, \dots, P_n be the n th row of intersections. How many people reach each point P_i ?

0140 Suppose A and B are two equally strong tennis players. Is it more probable that A will beat B in three sets out of 4 or in 5 sets out of eight?

AMNESIA - or even complete insanity.

The employees of the Utopia Factory have just woken from a long winter's slumber.

Their names are Alf, Bert, Charlie, Duggie, Ernie, Fred and George, but they not only don't know where they are, they also don't know who they are, what their occupations are or who their wives are.

But the subconscious works wonders. Certain remarks are made, all in the third person, and any remark which anyone makes in which his own name comes is false, while anything he says about anybody else is true.

Their occupations are, but not necessarily respectively, Door-Knob Polisher, Door-Opener, Door-Shutter, Bottle-Washer, Sweeper-Upper, Welfare Officer, and Worker. And their wives' names are Agnes, Beatrice, Clarissa, Diana, Ethel, Flossie and Gertie. No man has the same first letter to his name as his wife has. Below are 21 numbered remarks. Each man makes three, and the total of the numbers of each man's remarks is the same, except for Duggie and Fred who are each one out.

- (1) Ernie is the Worker.
- (2) Fred is not the Door-Opener.
- (3) Gertie is married to the Welfare Officer.
- (4) Clarissa's husband is the Door-Knob Polisher.
- (5) Bert is not the Door-Opener.
- (6) George is married to Diana.
- (7) Agnes is not married to Bert.
- (8) Duggie is the Worker.
- (9) Charlie's wife is Flossie.
- (10) Clarissa is married to Bert.
- (11) Duggie is married to Beatrice.
- (12) Alf is the Bottle-Washer.
- (13) Charlie is the Door-Opener.
- (14) George is the Door-Knob Polisher.
- (15) George is not the Door-Shutter.
- (16) Charlie is the Door-Shutter.
- (17) Ernie is the Welfare Officer.
- (18) Ernie is married to Gertie.
- (19) The numbers of two of Bert's remarks are perfect squares.
- (20) Charlie is the Bottle-Washer.
- (21) Fred is the Door-Opener.

Find out for each man his occupation, the name of his wife, and which remarks he makes.

(Answer p.28)