PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who send correct solutions by 30 June 1971, will be published in the next issue of Parabola.

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J141 The real numbers a, b and c are such that $a^2 + 4b^2 + 9c^2 = 2ab + 6bc + 3ca$. Prove that $a = 2b = 3c$.

J142 Prove that if n is an odd number then $n^4 - 18n^2 + 17$ is divisible by 64.

J143 One evening a number of people at a guest house decided to hire a bus for the next day and to share the cost equally. However, next morning two of the guests were unable to go, so the other members of the party were required to pay an additional 72 cents each. Just before the tour commenced a new guest arrived and joined the party, enabling 40 cents to be refunded to each of the other passengers.

What was the cost of hiring the bus?

0144 Prove that if the lowest common multiple of two numbers is equal to the square of their difference, then their highest common factor is the product of two consecutive integers.

0145 A, B and C are the angles of triangle ABC and a,b,c inches are the lengths of the corresponding sides. Prove that if cot A, cot B, and cot C are in arithmetic progression so are a^2, b^2 and c^2? Is the converse true?

0146 Let ABCD be any quadrilateral and construct squares on the sides (all lying outside the given quadrilateral). Let the two diagonals of the square on AB intersect at W, and similarly let X, Y and Z be the intersections of the diagonals of the other squares in order round the quadrilateral.

Show that the line segments WY and XZ are equal in length and perpendicular.

What can be deduced about the centre points of three squares on the sides of a triangle?

0147 The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents an ellipse whose axes of symmetry are the co-ordinate axes. Let a rectangle (with sides parallel to the axis) be inscribed in the ellipse. Find the maximum and minimum possible values of the area of this rectangle.

0148 Two ferryboats start at the same instant from opposite sides of a river, travelling on routes at rightangles to the banks. Each travels at a constant speed but one is faster than the other. They pass at a point 630 yards from the nearer bank. Both boats remain at the wharfs 57 minutes before starting back. On the return trips they meet 350 yards from the other bank. How wide is the river?

0149 Prove that the ratio of the sum of the medians of every triangle to its perimeter is constant in the interval $(\frac{3}{4},1)$ and that this is the smallest interval for which the statement is correct.

0150 Two partners, A and B, sold a number of articles, each article fetching as many dollars as there were articles. They divided up the proceeds as follows: first A took $10 then B took $10 and so on until, it being B's turn, fewer than ten dollars remained. How much money was A required to return to B so that each should receive half of the money?

DID YOU KNOW ...

Inside InFermation: $F(n) = 2^{2^n}+1$ is a Fermat number, where n is a positive integer. Fermat observed that F(1) = 5, F(2) = 17, F(3) = 257 and F(4) = 65,537 are prime, and conjectured that all the F(n) are prime. Euler discovered that F(5) = 64×6,700,417 and 150 years later in 1880 Landry found a factor 274,177 of F(6). In 1905 Western proved indirectly that F(7) is not prime, but he could not find any factors. F(7) was only finally factorised by a computer in Los Angeles in September 1970. 28 other Fermat numbers are known to be composite whilst none others are known to be prime.