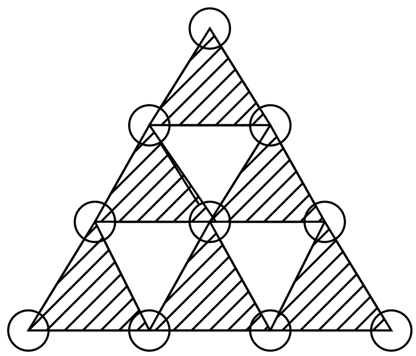
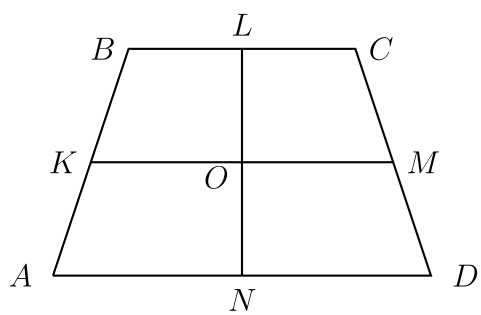
PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems,[1] accompanied by your name, school and year. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q1072 Is it possible to fill the empty circles in the diagram below with the integers $0, 1, \ldots, 9$ so that the sum of the numbers at the vertices of each shaded triangle is the same?



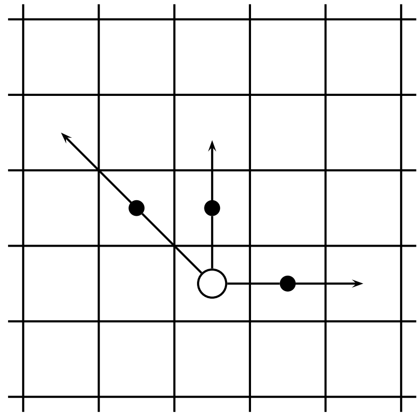
Q1073 The segments that connect the midpoints of the opposite sides of a convex quadrilateral $ABCD$ divide it into four quadrilaterals of the same perimeter. Prove that $ABCD$ is a parallelogram.



Q1074 Consider a closed path joining the 8 vertices of a cube which does not intersect itself. Prove that the path must include at least one edge of the cube.

Q1075 Fifty counters are placed on a 10×10 board so that 25 of them occupy the lower-left quarter of the board and the other 25 occupy the upper-right quarter. Each counter can jump over any neighbouring square to land on the next square (if the square it lands on is free). Is it possible that after several such moves these counters will occupy only the left half of the board? (The diagram below shows three possible moves for a white counter.)

[1]The problems this year have been taken from the Leningrad Mathematical Olympiads (1987-1991), collected by D. Formin and A. Kirichenko, and published by MathPro Press.



Q1076 Evaluate

$$\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cdots\cfrac{1}{2-\frac{1}{2}}}}}$$

where the digit 2 occurs 100 times.

Q1077 Does there exist a natural number n such that $n^n + (n+1)^n$ is divisible by 1999?

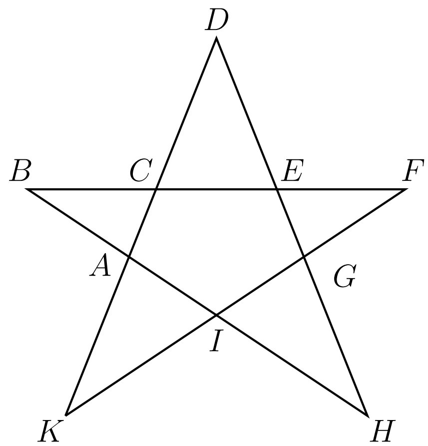
Q1078 All cells of a 3×3 table are occupied by zeros. One may choose any 2×2 subtable and increase all the numbers in the 4 cells by 1. Prove that one cannot obtain the table shown below using these operations.

4	9	5
10	18	12
6	13	7

Q1079 31 students play a game in which each student writes the numbers from 1 to 30 in any order. One of the students is designated the "leader". The leader compares his sequence with every other student's in turn and that student earns k points if his sequence and the leader's have exactly k numbers in the same position. It turns out that each of the 30 players earned a different number of points. Prove that one of the students' sequences was the same as the leader's.

Q1080 Each of the natural numbers $a,\ b,\ c$ and d is divisible by $ab - cd$, which is also a natural number. Prove that $ab - cd = 1$.

Q1081 Prove that it is impossible to draw a star (as shown below) so that the inequalities $AB < BC$, $CD < DE$, $EF < FG$, $GH < HI$ and $IK < KA$ all hold.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