WHO WON THE SYDNEY 2000 OLYMPIC GAMES?

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One of the focal points of the recent Sydney 2000 Olympic Games was the medal tally. We all want to know which country 'won' the Olympics or which country has the best athletes. One way to do this is to look at which country won the most medals. Throughout the period of the games regular updates were reported through the print media, television and the World Wide Web. The final tallies can be readily accessed in different formats from the official Olympic web site http://www.olympics.com and the Australian Bureau of Statistics web site http://www.abs.gov.au. The medal tallies drew attention to the performance of countries rather than just performances of individuals. So now that the tallies are complete how did the participating countries perform and which country won?

On the face of it the question of who won the Olympics is straightforward. From the data displayed in Table 1 and Table 2 it is immediately apparent that out of the 80 countries that received medals the USA was the clear winner receiving 97 of the 928 medals that were awarded. Australia came in fourth on this basis with 58 medals and of those that won medals Sri Lanka with a single bronze medal came equal last with ten others.

But a comparison based on numbers of medals alone is hardly fair. The USA has more than ten times the population of Australia from which to draw its athletes, a point which was often emphasized by the Australian media. The Australian Bureau of Statistics' Medal Tally, unlike the official Olympic Games Medal Tally, took into account the different populations. They divided the population of each country by the total number of medals won by that country and then ranked the results from smallest to largest. The final column in Table 1 shows the effects of population in an essentially equivalent way by listing the total number of medals that each country won per million of population. On this basis the Bahamas moves from the 57th position (with just two medals won) to the top position with 6.78 medals per million of population. Curiously Australia maintains its fourth position with 3.02 medals per million of population but the USA slips down to 46th position with 0.35 medals per million of population.

Population is just one factor that is important in ranking performances among the competing countries. There are many factors that could be taken into account. For example we could look at the worth of each medal itself. The official Olympic web site offered two ways to rank performance. One based on the total number of medals and the other based on medal value. The latter is determined by the number of Gold medals (or Silver medals for countries with equal numbers of Gold medals, or Bronze medals for countries with equal numbers of Gold and Silver). Interestingly the ranking by the

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Country	Population	Gold	Silver	Bronze	Total	Total per million of population
United States	275562673	39	25	33	97	0.352
Russia	146001176	32	28	28	88	0.603
China	1261832482	28	16	15	59	0.047
Australia	19169083	16	25	17	58	3.026
Germany	82797408	14	17	26	57	0.688
France	59329691	13	14	11	38	0.640
Italy	57634327	13	8	13	34	0.590
Cuba	11141997	11	11	7	29	2.603
Great Britain	59511464	11	10	7	28	0.470
Korea	47470969	8	9	11	28	0.590
Romania	22411121	11	6	9	26	1.160
Netherlands	15892237	12	9	4	25	1.573
Ukraine	49153027	3	10	10	23	0.468
Japan	126549976	5	8	5	18	0.142
Hungary	10138844	8	6	3	17	1.677
Belarus	10366719	3	3	11	17	1.640
Poland	38646023	6	5	3	14	0.362
Canada	31281092	3	3	8	14	0.448
Bulgaria	7796694	5	6	2	13	1.667
Greece	10601527	4	6	3	13	1.226
Sweden	8873052	4	5	3	12	1.352
Brazil	172860370	0	6	6	12	0.069
Spain	39996671	3	3	5	11	0.275
Norway	4481162	4	3	3	10	2.232
Switzerland	7262372	1	6	2	9	1.239
Ethiopia	64117452	4	1	3	8	0.125
Czech Rep	10272179	2	3	3	8	0.779
Kazakhstan	16733227	3	4	0	7	0.418
Kenya	30339770	2	3	2	7	0.231
Jamaica	2652689	0	4	3	7	2.639
Denmark	5336394	2	3	1	6	1.124
Indonesia	224784210	1	3	2	6	0.027
Mexico	100349766	1	2	3	6	0.060
Georgia	5019538	0	0	6	6	1.195
Lithuania	3620756	2	0	3	5	1.381
Slovakia	5407956	1	3	1	5	0.925
Algeria	31193917	1	1	3	5	0.160
Belgium	10241506	0	2	3	5	0.488
South Africa	43421021	0	2	3	5	0.115
Chinese Taipei	22191087	0	1	4	5	0.225

Table 1: Top 40 countries that won medals in the Sydney 2000 Olympics ranked by total numbers of medals won. Also listed are the population of each country and the total number of medals won per million of population.

Country	Population	Gold	Silver	Bronze	Total	Total per million of population
Morocco	30122350	0	1	4	5	0.166
Iran	65619636	3	0	1	4	0.061
Turkey	65666677	3	0	1	4	0.061
Finland	5167486	2	1	1	4	0.774
Uzbekistan	24755519	1	1	2	4	0.162
New Zealand	3819762	1	0	3	4	1.047
Argentina	36955182	0	2	2	4	0.108
DPR Korea	21687550	0	1	3	4	0.184
Austria	8131111	2	1	0	3	0.369
Azerbaijan	7748163	2	0	1	3	0.387
Latvia	2404926	1	1	1	3	1.247
Yugoslavia	14000000	1	1	1	3	0.214
Estonia	1431471	1	0	2	3	2.096
Thailand	61230874	1	0	2	3	0.049
Nigeria	123337822	0	3	0	3	0.024
Slovenia	1927593	2	0	0	2	1.038
Bahamas	294982	1	1	0	2	6.780
Croatia	4282216	1	0	1	2	0.467
Moldova	4430654	0	1	1	2	0.451
Saudi Arabia	22023506	0	1	1	2	0.091
Trinidad & Tobago	1175523	0	1	1	2	1.701
Costa Rica	3710558	0	0	2	2	0.539
Portugal	10048232	0	0	2	2	0.199
Cameroon	15421937	1	0	0	1	0.065
Colombia	39685655	1	0	0	1	0.025
Mozambique	19104696	1	0	0	1	0.052
Ireland	3797257	0	1	0	1	0.263
Uruguay	3334074	0	1	0	1	0.300
Vietnam	78773873	0	1	0	1	0.013
Armenia	3344336	0	0	1	1	0.299
Barbados	274540	0	0	1	1	3.642
Chile	15153797	0	0	1	1	0.066
Iceland	276365	0	0	1	1	3.618
India	1014003817	0	0	1	1	0.001
Israel	5842454	0	0	1	1	0.171
Kuwait	1973572	0	0	1	1	0.507
Kyrgystan	4685230	0	0	1	1	0.213
Macedonia	2041467	0	0	1	1	0.490
Oatar	744483	0	0	1	1	1.343
Sri Lanka	19238575	0	0	1	1	0.052

Table 2: Bottom 40 countries that won medals in the Sydney 2000 Olympics ranked by the total number of medals won. Also listed are the population of each country and the total number of medals won per million of population.

number of Gold medals rather than total number of medals has no effect on the order of the top five performing countries and Sri Lanka is still ranked equal last. However it does have significant implications for the Ukraine who is ranked 13th according to the total number of medals but 21st according to the number of Gold medals. Or consider Brazil who is ranked 22nd with 12 medals according to the total medal tally but is ranked 52nd after Mozambique who won a single Gold medal when it comes to the Gold medal ranking. So does a country perform better if it wins a single gold medal or if it wins no gold medals but ten silver medals? Clearly neither the total number of medals nor the number of Gold medals are entirely satisfactory for ranking performances. Gold, Silver and Bronze medals are not of equal value as suggested by the total medal tally but Silver and Bronze are not essentially worthless as suggested by the Gold medal tally. A Gold medal is worth more than a Silver which is worth more than a Bronze but how much more? Moreover are some Gold medals worth more than others? The difference between Gold and Silver may be a hundredth of a second or one hundred metres. One assessment of the relative worth of medals is provided by the Australian Olympic Committee who awarded a cash prize to athletes of $15,000 for each Gold medal, $7,500 for each Silver medal and $5,000 for each Bronze medal. This corresponds to relative scores of 6,3 and 2 respectively. We will adopt these relative values in the following.

So now we can assign different scores for different medals and we can take the different populations into account to arrive at overall comparison scores for each country. But this still neglects at least one important factor. Money! The population of Sri Lanka is similar to that of Australia but Sri Lanka only won one medal. We cannot infer from this that Australian athletes are really that much better than Sri Lankan athletes because Sri Lanka is a much poorer country than Australia. We might expect Australia to be able to offer its athletes better training facilities than Sri Lanka. Even more directly, a promising young athlete from Sri Lanka may be tempted to emigrate to Australia for a higher standard of living. The starting point for factoring in the influence of money is to identify a standard measure of the relative wealth of countries. One such measure is provided by the per capita Gross Domestic Product in terms of purchasing power parity. The Gross Domestic Product (GDP) is a measure of the amount of goods and services that a country produces in a given year. Dividing the GDP by the population gives the per capita GDP which gives a measure of how much on average each person is producing. When it comes to comparing per capita GDP across different countries it is necessary to have a common yard stick. One Australian dollar is not worth one US dollar. Pure currency conversions based on exchange rates are not very useful. You also need to take into account the relative cost of goods in local currencies within each country. For example one US dollar is worth about two Australian dollars based on exchange rates. However if you purchase a Big Mac burger in the US it will cost about $2.50 in US dollars and if you purchase a Big Mac burger in Australia it will cost about $2.50 in Australian dollars. On this basis the purchasing power of the Australian dollar and the US dollar are not two to one as the exchange rate suggests but they are about the same. The relative cost of Big Macs in local currencies provides a crude measure of the purchasing power

parities across different countries. As an aside this crude index is available from the web site for *The Economist,* http://www.economist.com. An official purchasing power parity index based on a whole basket of different goods is also available to more properly compare economic strengths of different countries. Our measure of the relative strengths of different economies is thus based on a measure of all the goods and services produced in the country per head of population (per capita GDP) in terms of standardized international dollar price weights (purchasing power parity). Fortunately this data is readily available courtesy of the CIA web site https://www.cia.gov/library/publications/the-world-factbook/. But how do we sensibly take this data into account in a revised ranking of performances in the Olympics? To what extent can a higher GDP be expected to boost performance? There is no simple answer to this question. Instead we will attempt to deduce an empirical power law relationship as was recently suggested by Robert Banks in his book "Towing Icebergs, Falling Dominoes and Other Adventures in Applied mathematics" (PUP, Princeton, 1998). This involves looking at the performance of each country compared to its GDP and and from this data attempting to develop our own assessment of to what extent the GDP helps a country win medals. Banks' proposal for scoring Olympics is a follows. To begin, as discussed above, assign different scores to each of the medals (Banks' suggests 3 points for Gold, 2 for Silver and 1 for Bronze but we will adopt the Australian Olympic Committee valuations of 6, 3 and 2) and tally up the scores for each country. The values, represented by S are shown in the third column in Tables 3 and 4. Now divide by the total population in millions to give a normalized score which we represent by S_0. These values are listed in the fourth column in Tables 3 and 4. To find an empirical power law relationship between the normalized score and the per capita GDP, which we represent by G_0, make a plot of $\log(S_0)$ versus $\log(G_0)$ for all countries who received medals. This gives a scatter of points and we can see a general trend showing that on average the higher the GDP the better the performance. We draw a straight line known as the 'line of best fit' through this scatter of points to give the desired empirical relationship. The resulting plot of the data points (based on the data in Tables 3 and 4) and the line of best fit to these data points is shown in Figure 1. A country whose data point on this plot is above the line of best fit has performed better than expected based on GDP and a country whose data point is below the line of best fit has done worse than expected. The equation for the line of best fit is

$$\log(S_0) = 0.6193 \log(G_0) - 5.3758 \tag{1}$$

which can be inverted by taking the exponential of both sides to become

$$S_0 = 0.004627 G_0^{0.6193}. \tag{2}$$

As an aside Banks carried out a similar analysis based on medal tallies for the Barcelona 1992 Olympic games using scores of 3 for Gold, 2 for Silver and 1 for Bronze and arrived at the empirical relationship $S_0 = 0.00182 G_0^{0.685}$, which is not so different from what we have found.

Equation (2) is the main result of this analysis. Given the per capita GDP of a country and its population we can use this relationship to determine how well we can ex-

Figure 1: Plot of the logarithm of the Score per million of population versus the logarithm of the per capita GDP with the straight line of best fit also shown

pect each country to perform; this is called the expected score which we denote by $\langle S \rangle = .004627 G_0^{0.6193} P / 10^6$. For example Australia has a per capita GDP of $G_0 = 22200$ and a population of $P = 19169083$ from which we deduce the expected score of 43. By contrast Sri Lanka with its lower per capita GDP but similar population has an expected score of 11.5. The USA has an expected score of 815. By comparing scores with expected scores it is easy to see that Australia did much better than the empirical expectation based on its population and economic status whereas the USA did much worse. France, Italy and Germany all performed pretty much as expected.

The final part of the analysis is to rank the performances of the different countries based on how they performed relative to expectations. In this part of Banks' analysis he calculated the difference between the obtained score and the expected score and ranked countries in decreasing order based on these differences. We do not entirely agree with this part of Banks' analysis as the differences are amplified by a country's population (better or worse depending on whether the difference is positive or negative respectively). For example if Country A scored 40 when their expected score was 20 Banks would give them a final score of 20 and if Country B scored 10 when they were expected to score 2 they would receive a score of 8 on this basis. However Country B has performed 5 times better than expected and Country A has only performed twice as well as expected. A fairer approach is to divide the difference in scores by

the expected scores. In which case Country A scores $\frac{40-20}{20} = 1$ and Country B scores $\frac{10-2}{2} = 4$. This index properly measures the relative performance with respect to the expected score which can then be compared across the different countries. The results of such an analysis for all countries who won medals in the Sydney 2000 Olympics are displayed in the final column of Table 3 and Table 4.

So the winner is Cuba!

Australia slips from fourth place to twelfth place and the USA slips from first place to 58th. To give some indication of Cuba's performance taking into account population and GDP, Australia would have had to win 125 additional Gold medals to beat Cuba.

There are many other factors that should be taken into account when ranking the performance of countries in the Olympics. For example a country situated in a very cold climate such as Sweden and Norway should not be expected to perform as well at a Summer Olympics. Norway's performance of 16th overall after taking into account the factors of population and GDP becomes even more favourable in this light. Perhaps you can create your own ranking system taking these and other considerations into account. If you do try this why not write in and tell us what you find. We'd love to hear from you.

Country	Per Capita GDP	Score S	Score per million	Expected Score $\langle S \rangle$	Performance $\frac{S - \langle S \rangle}{\langle S \rangle}$
Cuba	1700	113	10.1418	5.1627	20.88763762
Bahamas	20000	9	30.5103	0.6291	13.30604602
Jamaica	3350	18	6.7856	1.8709	8.621137472
Bulgaria	4300	52	6.6695	6.4183	7.101889954
Estonia	5600	10	6.9858	1.3878	6.205516238
Hungary	7800	72	7.1014	12.0687	4.965823061
Romania	3900	102	4.5513	17.3664	4.873416254
Lithuania	4800	18	4.9713	3.1907	4.641324063
Latvia	4200	11	4.5739	1.9511	4.637845659
Belarus	5300	49	4.7267	9.7137	4.044419638
Barbados	11200	2	7.2849	0.4089	3.891523512
Australia	22200	205	10.6943	43.6111	3.700636047
Georgia	2300	12	2.3907	2.8047	3.278582902
Slovenia	10900	12	6.2254	2.8229	3.250971329
Azerbaijan	1770	14	1.8069	3.6810	2.803294399
Norway	25100	39	8.7031	11.0004	2.545328871
Trinidad & Tobago	8500	5	4.2534	1.4758	2.388053295
Iceland	23500	2	7.2368	0.6513	2.070754718
Netherlands	23100	107	6.7328	37.0569	1.887449158
Russia	4200	332	2.2740	118.4497	1.802876386
Greece	13900	48	4.5276	18.0483	1.659526048
Kazakhstan	3200	30	1.7928	11.4715	1.615186532
Ukraine	2200	68	1.3834	26.7185	1.545052695
Slovakia	8500	17	3.1435	6.7892	1.503958529
Sweden	20700	45	5.0715	19.3309	1.327874445
Ethiopia	560	33	0.5147	14.9358	1.209455912
Moldova	2200	5	1.1285	2.4084	1.076061919
Croatia	5100	8	1.8682	3.9180	1.041849404
Kenya	1600	25	0.8240	13.5401	0.846361432
Denmark	23800	23	4.3100	12.6754	0.814539137
Czech Rep	11700	27	2.6285	15.7177	0.717806263
Yugoslavia	1800	11	0.7857	6.7208	0.636721878
New Zealand	17400	12	3.1416	7.4732	0.605732853
Switzerland	27100	28	3.8555	18.6946	0.497759762
Finland	21000	17	3.2898	11.3587	0.496648118
Oatar	17000	2	2.6864	1.4357	0.393024305
Poland	7200	57	1.4749	43.7774	0.30204062
Macedonia	3800	2	0.9797	1.5567	0.284778176
DPR Korea	1000	9	0.4150	7.2345	0.244034764
Korea	13300	97	2.0434	78.6374	0.233510403

Table 3: Top 40 countries that won medals in the Sydney 2000 Olympics ranked by medal score taking into account values of medals, number of medals, population and GDP.

Country	Per Capita GDP	Score S	Score per million	Expected Score $\langle S \rangle$	Performance $\frac{S-\langle S \rangle}{\langle S \rangle}$
France	23300	142	2.3934	139.0833	0.020970601
Italy	21400	128	2.2209	128.1758	-0.001371413
Germany	22700	187	2.2585	190.9867	-0.020874069
Costa Rica	7100	4	1.0780	4.1670	-0.04007573
Mozambique	1000	6	0.3141	6.3729	-0.058518871
Armenia	2900	2	0.5980	2.1571	-0.072833968
Uzbekistan	2500	13	0.5251	14.5652	-0.107460244
Great Britain	21800	110	1.8484	133.8771	-0.178350784
Austria	23400	15	1.8448	19.1119	-0.215150313
Kyrgystan	2300	2	0.4269	2.6179	-0.236020805
Cameroon	2000	6	0.3891	7.9025	-0.240749739
Uruguay	8500	3	0.8998	4.1857	-0.283267035
Canada	23300	43	1.3746	73.3305	-0.413614087
Algeria	4700	15	0.4809	27.1331	-0.447169947
Morocco	3600	11	0.3652	22.2130	-0.50479366
Belgium	23900	12	1.1717	24.3896	-0.507987538
Spain	17300	37	0.9251	77.9732	-0.525477918
United States	33900	375	1.3609	814.8404	-0.539787187
Kuwait	22500	2	1.0134	4.5275	-0.558255768
Ireland	20300	3	0.7900	8.1734	-0.632954869
Iran	5300	20	0.3048	61.4862	-0.674723541
Turkey	6200	20	0.3046	67.8067	-0.705043967
Chinese Taipei	16100	11	0.4957	41.3776	-0.734155705
China	3800	246	0.1950	962.1908	-0.744333443
South Africa	6900	12	0.2764	47.9070	-0.749514568
Nigeria	970	9	0.0730	40.3742	-0.777085151
Portugal	15300	4	0.3981	18.1538	-0.779660969
Japan	23400	64	0.5057	297.4520	-0.784839261
Argentina	10000	10	0.2706	51.3068	-0.805094024
Saudi Arabia	9000	5	0.2270	28.6450	-0.825449273
Sri Lanka	2600	2	0.1040	11.5975	-0.827549443
Israel	18300	2	0.3423	11.7932	-0.830410393
Brazil	6150	30	0.1736	177.6010	-0.831081989
Thailand	6400	10	0.1633	64.4818	-0.844917503
Colombia	6200	6	0.1512	40.9790	-0.853583499
Mexico	8500	18	0.1794	125.9810	-0.857121296
Indonesia	2800	19	0.0845	141.8699	-0.866074483
Chile	12400	2	0.1320	24.0368	-0.916794319
Vietnam	1850	3	0.0381	38.4628	-0.922002604
India	1800	2	0.0020	486.7762	-0.995891336

Table 4: Bottom 40 countries that won medals in the Sydney 2000 Olympics ranked by medal score taking into account values of medals, number of medals, population and GDP.