PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems,[1] accompanied by your name, school and year. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q1082 There are 25 people sitting around a table and they are playing a game with a deck of 50 cards. Each of the numbers $1, 2, \ldots, 25$ is written on two of the cards. Each player has exactly two cards. At a signal, each player passes one of his cards – the one with the smaller number – to his right-hand neighbour. This move is repeated. Prove that, sooner or later, one of the players will have two cards with the same numbers.

Q1083 A pile of 500 matches is placed on the table. Two players play the following game. In one turn, each player can take from the pile 1 or 2 or 4 or 8 $\ldots$ (any power of 2) matches. The player who cannot move loses. Assuming perfect play who will win this game?

Q1084 If x and y are numbers such that $0 \le x \le 1$ and $0 \le y \le 1$, prove that

$$\frac{x}{1+y} + \frac{y}{1+x} \le 1.$$

Q1085 Suppose that $abc = 1$ and $a + b + c = 1/a + 1/b + 1/c$. Prove that one of these real numbers is equal to 1.

Q1086 Consider the acute angled triangle ABC with $\angle BAC = 30°$. Draw the altitudes BB_1 and CC_1. Let B_2 and C_2 be the midpoints of AC and AB, respectively. Prove that the lines B_1C_2 and B_2C_1 are perpendicular.

Q1087 Find a 100 digit natural number with all digits nonzero that is divisible by the sum of its digits.

Q1088 Consider a stack of N coloured bricks. One may take some of them from the bottom of the stack and put them on the top, preserving their order. Then the stack must be turned over. Prove that the number of different arrangements of bricks in the stack that can be obtained by using such operations does not exceed $2N$.

Q1089 A total of 119 residents live in a apartment block with 120 apartments. We call an apartment impossible if there are 15 or more people living there. Every day, the

[1]The problems this year have been taken from the Leningrad Mathematical Olympiads (1987-1991), collected by D. Formin and A. Kirichenko, and published by MathPro Press.

inhabitants of an impossible apartment have a quarrel and all go off to other (distinct) apartments in the house. Is it true that eventually there will be no more quarrels?

Q1090 Find all integer solutions of the system of simultaneous equations

$$\begin{aligned} ab+cd &= -1 \\ ac+bd &= -1 \\ ad+bc &= -1. \end{aligned}$$

Q1091 Suppose that $a<b<c$. Prove that the equation

$$\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=0$$

has exactly two roots x_1 and x_2 and that they satisfy the inequality $a<x_1<b<x_2<c$.