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Problems 1581–1590

Parabola thanks Marian Maciocha for contributing the idea behind Problem 1581.

Q1581 Prove that all the numbers

$$\begin{gathered}50201905\\502019201905\\5020192019201905\\50201920192019201905\end{gathered}$$

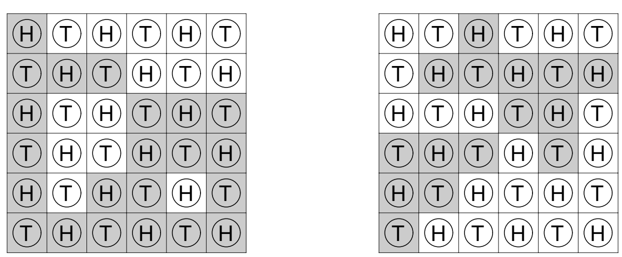
and so on are multiples of 13.

Q1582 Suppose that the following ratios are equal:

$$\frac{x+2y}{3x+5y} \quad \text{and} \quad \frac{x+3y}{4x+7y}$$

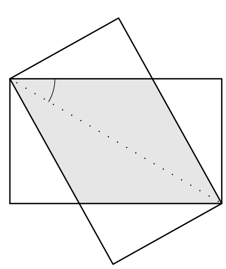
Find their value.

Q1583 Coins are placed in an $n \times n$ array, alternately heads up and tails up. A "move" means turning over some of the coins; the coins you turn must form one connected region (where "connections" may be horizontal or vertical, but not diagonal). For example, you can turn the coins indicated in grey in the left–hand diagram and it counts as one move, but to turn the coins in the right–hand diagram would require two moves.



How can you get all coins to face heads up in the minimum possible number of moves?

Q1584 The diagonal of an $a \times b$ rectangle, where $a \leq b$, is placed atop the "other" diagonal of an identical rectangle. Prove that the region of overlap is a rhombus, and find its area.



Q1585 Three questions are asked, and three answers given:

(A)	How many of these answers are correct?	**Answer:**	a
(B)	Of the first and last answers, how many are correct?	**Answer:**	b
(C)	Of the last two answers, how many are correct?	**Answer:**	c

Here, a is a number from $\{0, 1, 2, 3\}$, and b and c are numbers from $\{0, 1, 2\}$. From these answers it is possible to see that one of the answers is correct and another is incorrect; but it is impossible to tell whether the remaining answer is either correct or incorrect. What are the numbers a, b, and c? Which answer was correct, and which was incorrect?

Q1586 A display calendar for 2019 is arranged as shown. Each column is a fixed day of the week, starting with Monday; each month is a consecutive row of dates, starting on the appropriate day of the week, with gaps at the start if required.

	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T
Jan		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31					
Feb					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28					
Mar					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		
Apr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30							
May			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31				
Jun						1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30		
Jul	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31						
Aug				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31			
Sep							1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Oct		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31					
Nov					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			
Dec							1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

As you can easily count, this arrangement uses 37 columns. By starting on a different day of the week, could you use fewer columns than this?

Q1587 Let $f(x) = x^4 + ax^3 + bx^2 + cx + d$, and suppose that

$$d = \frac{c^2}{a^2} \,.$$

Find b in terms of a and c such that $f(x)$ is the square of a quadratic. Hence, find all solutions of the equation

$$x^4 + 2x^3 - 20x^2 + 6x + 9 = 0 \,.$$

Q1588 Yesterday, I tossed a coin 100 times. Today, I tossed the coin 101 times. Find the probability that I obtained more heads today than yesterday.

Q1589 A sequence of numbers $a_0, a_1, a_2, a_3, \ldots$ starts with $a_0 = 1$; each subsequent number in the sequence is given in terms of the previous one by the rule

$$a_n = a_{n-1}^2 - 2^{2^n+1}\,.$$

For example, you can check that

$$a_1 = -7\,,\ a_2 = 17\,, \ldots,\ a_6 = -32999768863368713983\,, \ldots.$$

Find a formula for a_n in terms of n.

Q1590 Jamie (nobody you have ever heard of, just a coincidence) is a chef, and has very regular habits for lunch. He has n different recipes; the day after cooking recipe 1, he always cooks recipe 2; the day after recipe 2 is recipe 3; and so on. The only thing that is different is that on the day after recipe n, he may cook either recipe 1 or recipe 2. If you know that Jamie starts with recipe 1 on day 1, what is the first day on which, as far as you know, Jamie could be cooking any one of his n recipes?