Evaluating the lucrative efficacy of properties in strategic Monopoly® gameplay

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1   Introduction

Monopoly® is a common household game where players attempt to bankrupt an opponent by buying properties, gaining profits, and taking control of the game board. Properties refer to the individual squares that are available for purchase and have individual independent variables such as property cost, house costs, and potential profit amounts. These properties are divided into property groups, organised by color and location. In clockwise order, those groups are the Brown, Light Blue, Pink, Orange, Red, Yellow, Green, and Navy with the Railroads and Utilities spread out; see Figure 1 which shows the board square locations of these properties.[2]

Four non-property squares are included [GO (Start), Jail (holding), Free (free), and Go to Jail (relocate)], and eight non-property squares are excluded from the figure for simplicity: two tax squares and three Chance and Community Chest squares. Players move forward at the roll of two dice, and gameplay movement is also influenced by other factors such Chance and Community Chest cards and a holding place in Jail.

Figure 1: A simplified Monopoly® board for reference. The numbers correspond with those in Table 1. Colors indicate property groups

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[2]Monopoly® is available in a large number of equivalent editions, featuring the same game rules and board layout but different property names and decoration. The USA Edition of the game is used here.

Table 1: The list of Monopoly® properties corresponding to Figure 1.

Number	Property Name	Number	Property Name
1	Go (Start)	17	Free (free)
2	Mediterranean Avenue	18	Indiana Avenue
3	Baltic Avenue	19	Kentucky Avenue
4	Reading Railroad	20	Illinois Avenue
5	Oriental Avenue	21	B+O Railroad
6	Vermont Avenue	22	Atlantic Avenue
7	Connecticut Avenue	23	Ventnor Avenue
8	Jail (holding)	24	Waterworks
9	St. Charles Place	25	Marven Gardens
10	Electric Company	26	Go to Jail (relocate)
11	States Avenue	27	Pacific Avenue
12	Virginia Avenue	28	Pennsylvania Avenue
13	Pennsylvania Railroad	29	North Carolina Avenue
14	St. James Place	30	Short Line Railroad
15	Tennessee Avenue	31	Park Place
16	New York Avenue	32	Boardwalk

Various strategies have been proposed for the game but none have been mathematically proven to be superior to the others [1]. There are four components that a player must consider when purchasing a property in a Monopoly® game: probability, long-term profit, short-term profit, and expected value. Probability refers to how frequently a square is landed on by an opposing player. Short-term profit refers to the profit a player could earn from a property with a hotel (the highest level of house per property) with the investment costs subtracted. Long-term profit refers to how quickly a property will recoup its investment costs to gain profit later in the game. Expected value refers to the expected amount of money a player could earn per rotation around the board. A balanced combination of these factors would mean that a property is more beneficial to the player owning it, making it a better fit for a winning strategy [2].

The aim of this study is to model and analyse which properties contribute best to a winning strategy.

Methodology

Probability

In a game of Monopoly®, players roll a pair of dice and move the corresponding number of spaces determined by the sum of rolls. In addition, a regulation rule states that if a player rolls "doubles" (the same number on both dice), then they can roll again after moving. This rule skews the probability distribution of the dice rolls but allows for a general pattern that could be repeated in a model of the game. The probabilities of game events depend on the rolls of the dice and on the squaree presently occupied by players. The probability that a player is on any given property is approximately,

$$P = \frac{2}{36}(3b + 11b + 10b + 4b) + \frac{4}{36}(5b + 9b + 8b + 6b) + \frac{6}{36}7b,$$

where P is the probability of landing on the squares in question and where b is the number of spaces between the square in question and one just previously landed on. For example, a player might need to roll a dice total of $b = 7$ to move from their present square to a given square. This equation only covers when a player lands and stays on a square; it does not count additional rolls for doubles. Chance and Community Chest cards contribute additional effects. While they do not apply to every property, the added probability they do bring can be shown to be

$$P_A = P_E + \frac{1}{16}C_p,$$

where P_A is now the probability of the square with the additional probability added from landing on the wild card square, P_E is the existing dependent probability determined from the first equation, and P_C is the likelihood of landing on Chance or Community Chest. Finally there is also the matter of Jail, a holding place that a player must visit from either rolling doubles thrice, drawing a Go to Jail card, or landing on the Go to Jail square. The probability of this happening is

$$P_J = \frac{1}{216} + \frac{3}{16}P_C + P_E.$$

Short-term profit

In order for a player to buy properties, they need to make smart investments but also keep liquidated money to pay fees and buy properties. The utility of a property can be judged by how much money it will provide quickly. A player must first buy the property, then develop it in order to build Houses and an eventual Hotel, the most profitable development option. The equation for the amount of money provided by a hotel on a property can be written as

$$E = S_p - (C_p - C_h)$$

where E is the amount that a player will earn when the property is first landed on by an opposing player, S_p is the potential profit of the property with a Hotel, C_p is the initial cost of the property, and C_h is the cost of a hotel.

Long-term profit

In addition to nearly immediate profit, a player must also consider the amount of money that a property will yield in a long game. It takes a certain number of rolls to recoup the money invested and that number of rolls is dependent on the number of houses on a property. As more houses are added, the number of rolls decreases, modeled as an exponential decay:

$$S_X = C_p \times r^X$$

where S_X refers to the amount of profit for a given number of houses ranging from zero to seven, r is the rate at which the time to recoup decreases and X is the development stage of the property, from one to seven; see Table 2. A slow rate r, close to the value 1, means that the player must conduct more rolls in order to recoup the property cost. A value for the rate h can be found empirically by plotting the profit level Y associated with a given property's developmental stage X.

Table 2: The seven property developmental stages

X	Property development at stage X
1	Property owned alone
2	Property owned with group
3	Property with 1 house
4	Property with 2 houses
5	Property with 3 houses
6	Property with 2 houses
7	Property with hotel

Expected value

Expected value is the fourth factor to consider when buying property. It is the mean profit value for a property weighted by probability. Here, that is the average sum of money that a player will earn each rotation around the board:

$$V = P \times S_X \,,$$

where V is the expected value of a property. This formula applies to short-term profit, meaning within the first rotation while owning the property in question.

Results

Individual results

Based on the calculations performed in each of the four categories, there are certain properties that yield better results in terms of probability and profit.

Overall, the four categories were weighted equally in order to create a complete ranking system; Figure 3.

Table 3: The individual Monopoly® properties ranked by usefulness to a player.

Rank	Property	Probability	Profit Rate	Short Profit	EV
1	New York	3.06	0.5981	300	10.838
2	St. James	2.79	0.5974	270	7.533
3	Tennessee	2.94	0.5974	220	6.4668
4	Vermont	2.67	0.5587	200	5.3464
5	Connecticut	2.44	0.5679	230	5.612
6	Oriental	2.33	0.5587	200	4.66
7	Boardwalk	2.48	0.6465	600	14.8998
8	Virginia	2.45	0.5959	240	5.8884
9	Atlantic	2.76	0.6277	140	3.857
10	Marvin Gardens	2.60	0.6266	170	4.36577
11	Ventnor	2.69	0.6277	140	3.76928
12	BO RR	2.962	0.5	0	0
13	Illinois	3.149	0.6288	110	3.4639
14	Park Place	2.26	0.6519	150	3.3829
15	Pennsylvania RR	2.902	0.5	0	0
16	Reading RR	2.901	0.5	0	0
17	States	2.36	0.6029	110	2.5909
18	St. Charles	2.31	0.6029	110	2.5399
19	Electric	2.74	0.4	28	0.7668
20	Indiana	2.89	0.6296	80	2.31848
21	Waterworks	2.57	0.4	28	0.72801
22	Kentucky	2.78	0.6296	80	2.224
23	Baltic	1.89	0.5803	140	2.65999
24	Short Line RR	2.49	0.5	0	0
25	Pennsylvania	2.31	0.6414	80	1.8481
26	N. Carolina	2.39	0.6484	-25	-0.5976
27	Pacific	2.25	0.6484	-25	-0.5636
28	Mediterranean	1.79	0.5738	-60	-1.074

Properties ranked closer to "1" are deemed more beneficial to own. There are many interesting patterns to note. All of the Railroad properties (properties tagged with RR) have short-term profit values of 0 since the cost to buy and develop them is equal to

the profit to be gained from them. Some properties even have negative profit values pointing to debt rather than profit in a short game. This ranked list of properties can best be seen as a way to prioritize the purchase of each square.

Property group results

As a generalisation, buying property groups in their entirety produces higher profits than the individual squares. Therefore, it is more financially beneficial to prioritise the buying groups over individual squares. To do so, the group rankings were organised on the median rank of the squares in that group based on individual performance; see Table 4.

Table 4: The Monopoly® property groups ranked by usefulness to an owning player.

Rank	Color	Med rank	Properties
1	Orange	2	New York, Tennessee, St.James
2	Light blue	4	Vermont, Connecticut, Oriental
3	Yellow	10	Atlantic, Ventnor, M. Gardens
4	Navy	10.5	Boardwalk, Park Place
5	Railroads	15.5	B+O, Reading, Pennsylvania, Short Line
6	Pink	17	Virginia, States, St. Charles
7	Red	20	Illinois, Indiana, Kentucky
8	Utility	20	Electric, Waterworks
9	Brown	25.5	Baltic, Mediterranean
10	Green	26	Pennsylvania, N.Carolina, Pacific

Groups closer to "1" are considered more beneficial to own. "Med Rank" refers to the median of the rankings of a group's individual properties in Table 3. A tie for 7th was broken by mean. As with the individual rankings, this group list serves as a method to effectively prioritize the purchase of properties in gameplay. What this list (in addition to the individual rankings list) suggests is that some groups do benefit a player more than others. The difference in results in the four categories as well as the large gaps in median ranks between the groups account for this. Based on these lists, it can be recommended that the Orange, Light Blue, and Yellow are the most beneficial options while the Brown and Green are the worst. The rest of the groups can be bought as filler properties as to continue to outnumber an opponent but should not be prioritized over the top three.

2 Discussion

The game of Monopoly® is complex and leaves much open to the interpretation of the player at the moment. Taking this into consideration, the research presented here only concerns games with two players under regulation game rules. The adjustments of "house rules" were not applied and could provide nearly endless variations of the game where different strategies could work better. This endless variation can be seen in the fact that no property or property group excelled in every category, though some were the worst. Strategy largely depends on the conditions and rules of the game at the time and no strategy (including the one suggested in Section 3) is foolproof. Future research into the topic would be beneficial, including the implementation of a strategy in larger, longer games or in simulation.

Acknowledgements

References

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[2] S.D. Abbott and M. Richey, Take a walk on the Boardwalk, *College Mathematics Journal* **28** (1997), 162–171.