Parabola Volume 55, Issue 3 (2019)

Problems 1601–1610

Parabola would like to thank Shiva Oswal for contributing Problems 1603 and 1604, and Sin Keong Tong for Problem 1605.

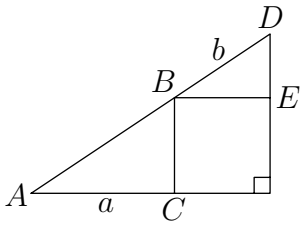
Q1601 Find conditions on a, b, c, d such that the curve

$$y = f(x) = x^4 + ax^3 + bx^2 + cx + d$$

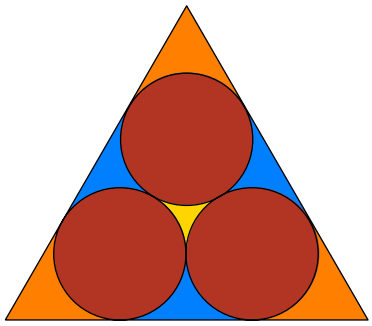
has a double tangent.

Q1602 A jar contains r red marbles and b blue marbles. If two of them are picked at random without replacement, then the probability that both the marbles drawn are red is 0.51, while probability that both are blue is 0.07. Find r and b.

Q1603 A right–angled triangle has a square drawn in the corner as shown. Find the area of the square in terms of the lengths $a = AC$ and $b = BD$.



Q1604 A peculiar dartboard with a peculiar scoring system consists of an equilateral triangle with three inscribed circles of equal radius within.



The brown area is worth x points, the orange area is worth 3 points, the blue area is worth 9 points, and the gold area is worth 15 points. Assuming that a "hit" on this board has an equal chance of landing anywhere, what must the value of x be if the expected value of a "hit" is $6\sqrt{3} - 3$ points?

Q1605 Calculate the constant term when the expression

$$\left(1 + x + \frac{1}{x}\right)^{10}$$

is expanded and like terms collected.

Q1606 In the following 4×4 array of numbers,

3	3	3	2
1	2	2	3
3	2	2	3
1	1	2	1

find a path which includes every square and does not visit any square more than once. You may start in any square you wish and finish in any square you wish; from a square containing the number n you must move to a square which is n squares away (either horizontally, vertically or diagonally).

Q1607 How many triples (x, y, z) of positive integers satisfy the conditions

$$x < y < z \quad \text{and} \quad x + xy + xyz = 2019\,?$$

Q1608 Define a decimal x by concatenating the digits in the sequence of perfect squares,

$$x = 0 \cdot 149162536496481100121 \cdots .$$

Prove that x is not a rational number.

Q1609 Simplify the sum

$$\sec 0 \sec 1 + \sec 1 \sec 2 + \sec 2 \sec 3 + \cdots + \sec(n-1) \sec n\,.$$

Q1610 Suppose we have a bag containing n different objects. We wish to choose a collection of objects A from S: the collection A may contain all the objects in S, or none, or anywhere in between. Then we want to choose two collections B and C from the objects in A: these, again, may contain any number of objects from A, except that there must be at least one object which is in both B and C. In how many ways can the choice of the three collections A, B, C be made?