Parabola Volume 58, Issue 1 (2022)

Magnetic field strength below a three-phase power line

Rachael Wen[1]

1 Introduction

Hello, my name is Rachael. The topic of my paper is magnetic fields, which is a kind of invisible radiation that is emitted by all electrical devices. In everyday life, cell phones, microwaves, computers, televisions and power lines all emit magnetic fields. Many may view magnetic fields with hesitance because they believe that magnetic fields are harmful to human health. So far, there is no published, large-scale and widely-accepted research showing the safe level of magnetic field exposure for humans. Per [1], only a handful of developed countries have legal requirements limiting the strength of magnetic fields: France requires the exposure of children to magnetic fields of $1\,\mu$T to be avoided; Germany requires the magnetic fields emitted from facilities greater than 1 kV to be minimized; Luxembourg recommends that the construction of new living spaces in the immediate vicinity of overhead power lines to be avoided. In some developed countries, such as Australia, India and the United States, there are no laws limiting the level of magnetic field a device can emit. This means that people can be exposed to limitless levels of magnetic fields. The ambiguity surrounding the effects of magnetic fields may be what makes the subject so frightening. Until there is more definitive research, people may want to limit their exposure to magnetic fields. Some ways to limit exposure include using a Bluetooth headset, watching less television and standing away from microwaves. These are all personal ways to reduce magnetic field exposure, and can be relatively easily implemented. On the other hand, one source of magnetic field that people will have a tough time avoiding is the ubiquitous power line. Unlike with electrical appliances, you cannot just turn off a power line. Except during a power outage, power lines are always on, meaning that power lines are a near constant source of magnetic fields. Most people do not seem too concerned about the magnetic field emitted from distribution power lines. Distribution power lines are the wires that are usually supported 10 meter or so above the ground on wood poles. But people who live near transmission power lines seem to have an increased concern about magnetic fields. Transmission power lines, shown in Figure 1, are the wires that are supported on steel towers that are usually 20 m to 50 m tall. Compared to distribution power lines, transmission power lines usually carry a lot more electrical current and, therefore, has the potential to create much stronger magnetic fields. Out of concern for the public, I decided to calculate the strength of the magnetic field that a person may experience if he or she were to stand below a set of transmission power lines. My paper only uses algebra and trigonometry.

[1]Rachael Wen is a senior student at San Marino High School, San Marino, California, USA.

Figure 1: A 3-phase transmission circuit. Source: Wikimedia Commons

2 Background Literature

Before I tried to solve this problem on my own, I searched the Internet. I found three research papers that described how to calculate the magnetic field strength below power lines. The three papers are [2], [3] and [4]. Strangely, [2] and [4] share two authors and [3] and [4] also share two authors. So, it may be that these three papers are, in fact, one paper published three times. The paper that I focused on the most is [2], which generally describes the process that I am looking for. However, [2] uses complicated math notation that can confuse young readers. So, I decided to write my own research paper that describes the mathematical process using only algebra and trigonometry. Because [2] provides a numerical example, I can repeat the same example and compare answers to see if my math is correct.

3 Mathematical Strategy

A transmission circuit usually consists of three bundles of conductors, where each bundle carries one phase of power; this means that the electric current phase angle for each bundle is different. A bundle usually consists of one, two or four conductors. If you look carefully at Figure 1, then you will see that that the transmission towers support three bundles of conductors, where each bundle consists of a pair of conductors. However, within a bundle, the separate conductors are very closely spaced together so that a bundle of conductors can be modeled as one conductor. In Figures 2, 3 and 4, I have drawn a simplified diagram of a hypothetical transmission circuit in the XYZ, YZ and XZ coordinate planes, respectively. The three bundles of conductors are represented by a red conductor (Conductor 1), a blue conductor (Conductor 2) and a purple conductor (Conductor 3).

In Figure 2, I have drawn a person standing underneath the power lines. We want to calculate the magnetic field strength at the location where the person is standing. To do this, I will use the four-part strategy listed in Figure 5: (1) As a starting point, we will only look at one conductor; (2) approximate the shape of the conductor using small, straight segments; (3) calculate the magnetic field strength due to just *one* of these segments; (4) define the position of the segment by using its left endpoint. By following this four-part strategy, we can find the magnetic field strength at the location of the person due to just one segment of one conductor. We can then expand the process to find the magnetic field strength due to all the segments of the conductor. We can then further expand the process to include the other two conductors.

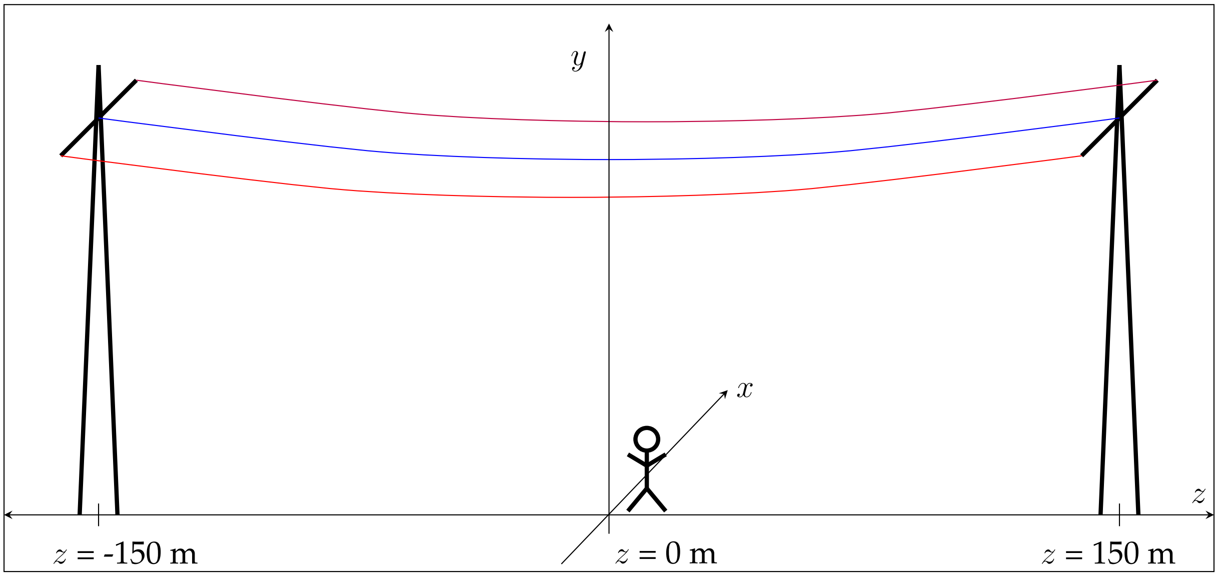

Figure 2: Depiction of a 3-phase circuit on the XYZ-plane.

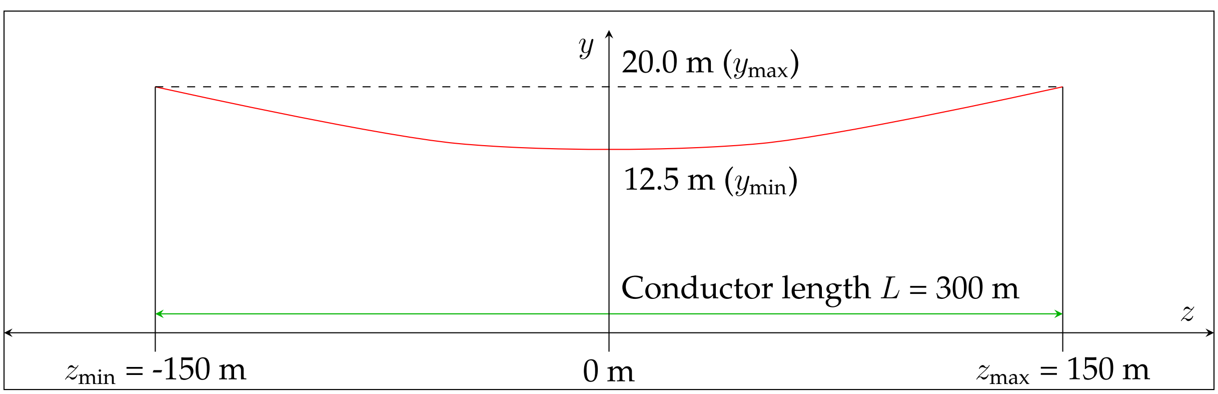

Figure 3: Depiction of a 3-phase circuit on the YZ-plane. In this scenario, the two conductor attachment points are 20 m above ground and its sag point is 12.5 m above ground. Note: this diagram only shows the red set of bundle conductors because, viewing the conductors in this coordinate plane, the red set of bundle conductors block the view of the blue and purple sets of bundle conductors.

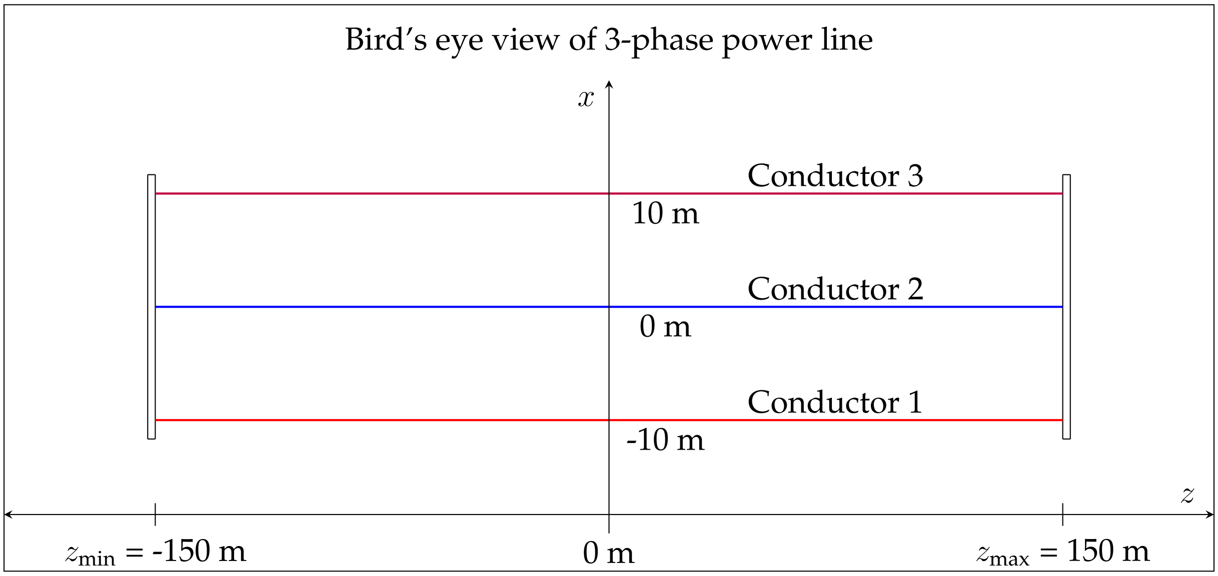

Figure 4: Depiction of a 3-phase circuit on the XZ-plane.

4 Find the Magnetic Field Created by a Conductor

For a given conductor, we will divide its length into m segments and use n to represent the index of the segments. The position of any Segment n will be expressed as (x_n, y_n, z_n), as shown in Figure 5. Since Segment n has finite length, there is no single point that can truly represent its position. But if we make the segment relatively short, then we can pretend that a single point can represent its position. This single point can be anywhere along the length of Segment n but some easy choices would be the left endpoint, midpoint, or right endpoint of the segment. In this paper, I will use the left endpoint to represent the position of Segment n. Based on Figure 4, we see that x_n = -10 for all segments of Conductor 1, $x_n = 0$ for all segments of Conductor 2, and x_n = 10 for all segments of Conductor 3. Finding z_n is easy if we divide the conductor length L into m equal segments, where L is approximated as the straight line distance between the two towers as shown in Figure 3. The value z_n is then calculated using the following equation:

$$z_n = z_{\text{min}} + \frac{L(n-1)}{m} . \tag{1}$$

In (1), z_{min} is the leftmost z-value of the conductor as shown in Figure 3. The only coordinate left to find is y_n. Based on Figure 3, we see that when a conductor is installed, it has a curved shape that resembles a parabola. This shape is called a catenary. The y-value of the conductor, which represents its height above ground, can be found using the catenary curve formula:

$$y = y_{\text{min}} \cosh\left(\frac{z}{a}\right) . \tag{2}$$

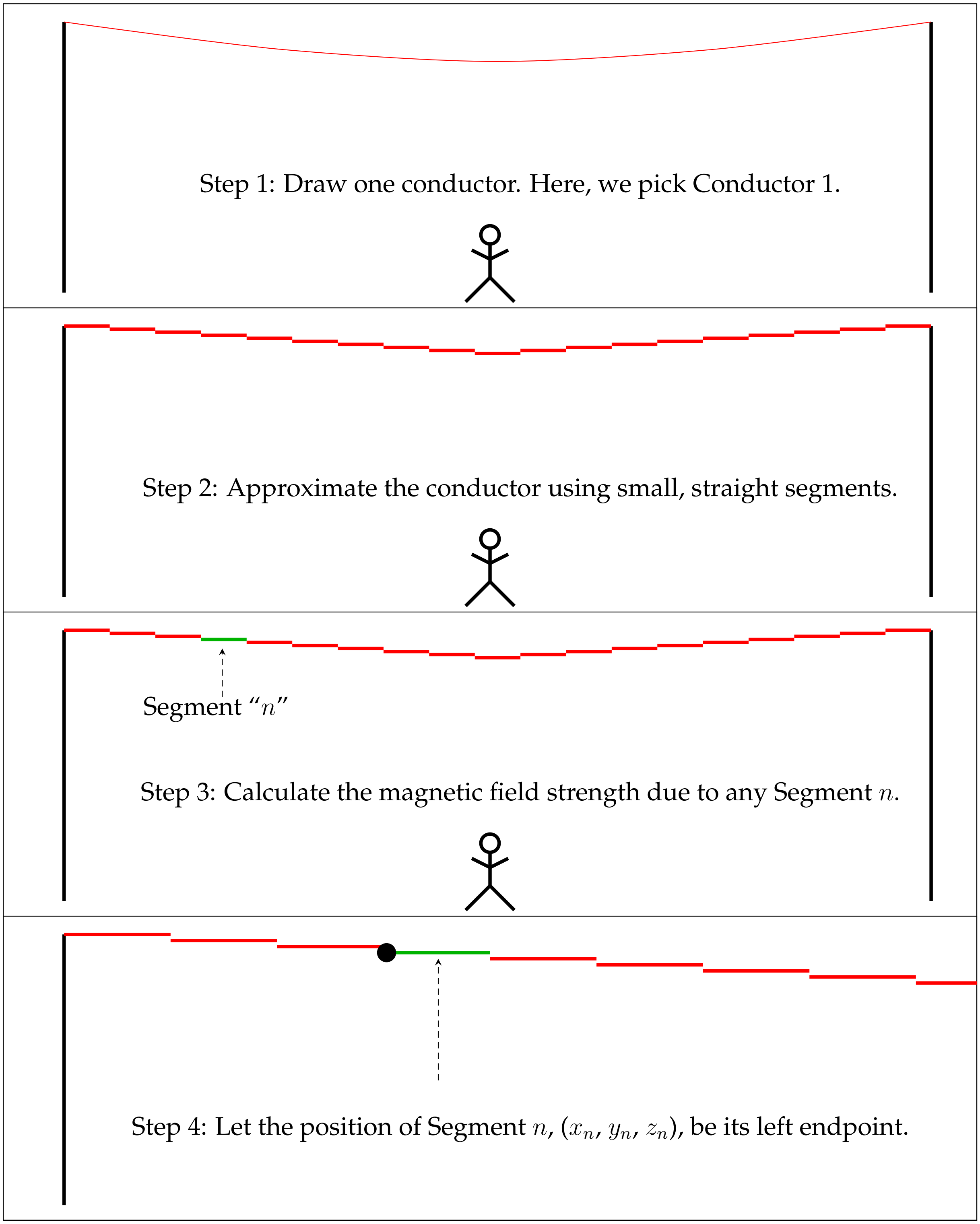

Figure 5: Our strategy: Calculate the magnetic field strength due to just one segment of one conductor.

In (2), y_{min} is the conductor sag point as shown in Figure 3 and a is the conductor radius of curvature. We can find a using this formula:

$$a = \frac{0.5L}{\cosh^{-1}\left(\dfrac{y_{\text{max}}}{y_{\text{min}}}\right)} \tag{3}$$

In (3), y_{max} is the conductor attachment point as shown in Figure 3. So, for every value z_n, we can use (2) to find the corresponding value y_n. Now that we have the coordinate of Segment n, the next step is to find the magnetic field strength that Segment n creates at the location where the person is standing. This magnetic field strength can be found using the Biot Savart Law:

$$dB_n = \frac{\mu_o I \overrightarrow{dL_n} \times \hat{r}_n}{4\pi {r_n}^2} \, . \tag{4}$$

This formula looks very complicated! But we can break down what it means. Since the length of Segment n is relatively tiny compared to the total lengths of all three conductors, Segment n will only contribute a tiny magnetic field at where the person is standing. The variable dB_n represents this tiny contribution (the "d" in the variable stands for *differential*, which indicates that the quantity is tiny). μ_o is the magnetic permeability of free space and has the value $4\pi \cdot 10^{-7}$ H/m. I is the amount of electrical current in Segment n in Amperes, which will be the same as the amount of current in the conductor. Because I is a complex number, it will be given in the form of a magnitude and phase angle, i.e., $I = |I|\angle\theta°$. For math purposes, we need to rewrite I in standard form:

$$I = |I|\angle\theta° = I(\cos\theta + j\sin\theta) \, . \tag{5}$$

Next, r_n is the distance between Segment n and the person. If we let the position where the person is standing be (x_p, y_p, z_p), then we can find r_n by using the distance formula:

$$r_n = \sqrt{(x_p - x_n)^2 + (y_p - y_n)^2 + (z_p - z_n)^2} \, . \tag{6}$$

The variable $\overrightarrow{dL_n}$ represents a vector because it has an arrow on top of it. A vector is a quantity that includes both magnitude and direction (e.g., "50 miles per hour to the West" is a vector). Specifically, $\overrightarrow{dL_n}$ is the vector representing the length and direction of Segment n as shown in Figure 6. Since the overall conductor length is approximately L and we divide the conductor into m segments, the length Δz of any Segment n will be:

$$\Delta z = \frac{L}{m} \, .$$

As for the direction of Segment n, we see that, based on Figures 2 through 5, all conductor segments will be pointing perfectly in the z-direction. So, $\overrightarrow{dL_n}$ can be rewritten like this:

$$\overrightarrow{dL_n} = \Delta z \cdot \hat{z} \, . \tag{7}$$

In the above equation, $\hat{z}$ is the way we explicitly show that Segment n is pointing in the z-direction. Finally, the variable $\hat{r}_n$ represents a unit vector because it has a "hat" shape on top of it. If you remember from above, the variable r_n represented the distance between Segment n and the person. So, $\hat{r}_n$ must represent the unit vector pointing from Segment n to the person as shown in Figure 6:

$$\hat{r}_n = \frac{(x_p - x_n)\hat{x} + (y_p - y_n)\hat{y} + (z_p - z_n)\hat{z}}{r_n}\,.$$

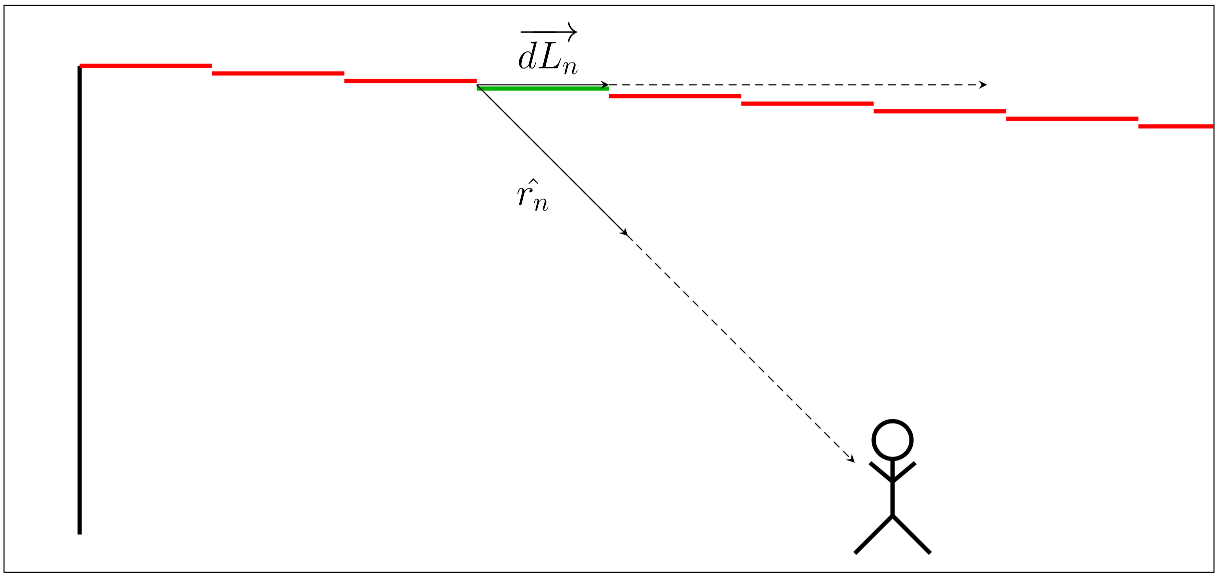

Figure 6: Diagram showing the vector dL_n and unit vector r_n.

Now we know every variable in (4)! To solve for (4), the last thing we have to understand is how to perform the cross product $\overrightarrow{dL_n} \times \hat{r_n}$ in the numerator. As it turns out, finding the cross product of two vectors is pretty straightforward:

$$\overrightarrow{dL_n} \times \hat{r_n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \Delta z \\ \frac{x_p - x_n}{r_n} & \frac{y_p - y_n}{r_n} & \frac{z_p - z_n}{r_n} \end{vmatrix}$$

$$= \frac{-\Delta z(y_p - y_n)}{r_n}\hat{x} + \frac{\Delta z(x_p - x_n)}{r_n}\hat{y} \tag{8}$$

We can substitute (8) into (4) to get a simpler looking equation:

$$dB_n = \frac{-\mu_o I \Delta z(y_p - y_n)}{4\pi {r_n}^3}\hat{x} + \frac{\mu_o I \Delta z(x_p - x_n)}{4\pi {r_n}^3}\hat{y}\,. \tag{9}$$

Looking at (9), we see that Segment n creates, at the location where the person is standing, a differential magnetic field in the x-direction and a differential magnetic field in the y-direction. The magnetic fields due to Segment n can be separated out according to their direction:

$$dB_{x,\,n} = \frac{-\mu_o I \Delta z (y_p - y_n)}{4\pi {r_n}^3}\hat{x} \tag{10}$$

$$dB_{y,\,n} = \frac{\mu_o I \Delta z (x_p - x_n)}{4\pi {r_n}^3}\hat{y} \tag{11}$$

After you solve (10) and (11), you will have the differential magnetic field in the x-direction and y-direction, respectively, created by just one segment of one conductor. But, to find the total magnetic field due to the whole conductor, we need to solve (10) and (11) for all m segments. The total magnetic field in the x-direction, B_x, and the total magnetic field in the y-direction, B_y, due to the conductor can be written using summation notation:

$$B_x = \sum_{n=1}^{m} dB_{x,\,n} \tag{12}$$

$$B_y = \sum_{n=1}^{m} dB_{y,\,n} \tag{13}$$

For example, if we break the conductor into m = 300 segments, then (12) will have 300 terms and (13) will also have 300 terms.

5 Find the Magnetic Field Created by Three Conductors

Let the magnetic field in the x and y direction due to Conductor 1 be $B_{x,\,\text{Cond 1}}$ and $B_{y,\,\text{Cond 1}}$, respectively. Similarly, let the magnetic field in the x and y direction due to Conductor 2 be $B_{x,\,\text{Cond 2}}$ and $B_{y,\,\text{Cond 2}}$, respectively. Finally, let the magnetic field in the x and y direction due to Conductor 3 be $B_{x,\,\text{Cond 3}}$ and $B_{y,\,\text{Cond 3}}$, respectively. In this case, the total magnetic field in the x and y direction due to all three conductors, which we can call $B_{x,\,\text{total}}$ and $B_{y,\,\text{total}}$, respectively, will be:

$$B_{x,\,\text{total}} = B_{x,\,\text{Cond 1}} + B_{x,\,\text{Cond 2}} + B_{x,\,\text{Cond 3}} \tag{14}$$

$$B_{y,\,\text{total}} = B_{y,\,\text{Cond 1}} + B_{y,\,\text{Cond 2}} + B_{y,\,\text{Cond 3}} \tag{15}$$

The magnitude of the overall magnetic field B at the location where the person is standing will be the resultant of $B_{x,\,\text{total}}$ and $B_{y,\,\text{total}}$. Because the current I is a complex number, $B_{x,\,\text{total}}$ and $B_{y,\,\text{total}}$ will also be complex numbers. So, the magnitude of B will be:

$$|B| = \sqrt{\Big(\text{Re}\left(B_{x,\,\text{total}}\right)\Big)^2 + \Big(\text{Im}\left(B_{x,\,\text{total}}\right)\Big)^2 + \Big(\text{Re}\left(B_{y,\,\text{total}}\right)\Big)^2 + \Big(\text{Im}\left(B_{y,\,\text{total}}\right)\Big)^2}\,. \tag{16}$$

6 Numerical Example

The numerical example in this section comes from [2] and is based on the diagram in Figure 7 which is taken from [2]. Assume that there is a transmission circuit that consists of three bundles of conductors. The length of the conductors in the z-direction, the placement of the conductors in the x-direction, and the curvature of the conductors in the y-direction are exactly as depicted in Figures 2 through 4. Conductor 1 has a current of I = 1920 $\angle$ -30°; Conductor 2 has a current of I = 1920 $\angle$ -150°; and Conductor 3 has a current of I = 1920 $\angle$ 90°. Assume that a person is allowed to stand anywhere on the x-axis from x = -30 m to x = 30 m. What is the magnetic field strength at the location where the person is standing? The problem asks that we calculate the magnetic field strength at a height of 1 m above ground which is approximately halfway up the height of an average person. This means that we need to find the magnetic field strength for a range of coordinates, starting at (x_p, y_p, z_p) = (-30, 1, 0) and ending at (x_p, y_p, z_p) = (30, 1, 0). Based on my mathematical process, I created a Python computer code to solve the problem. My code generated the plot shown in Figure 8. In my plot, the independent variable is the x-coordinate in meters and the dependent variable is the magnetic field strength $|B|$ in Tesla. The answer published by the authors of [2] is reproduced in Figure 9. If we compare the two figures, we see that the results are very similar. A copy of my Python code is included in Appendix A.

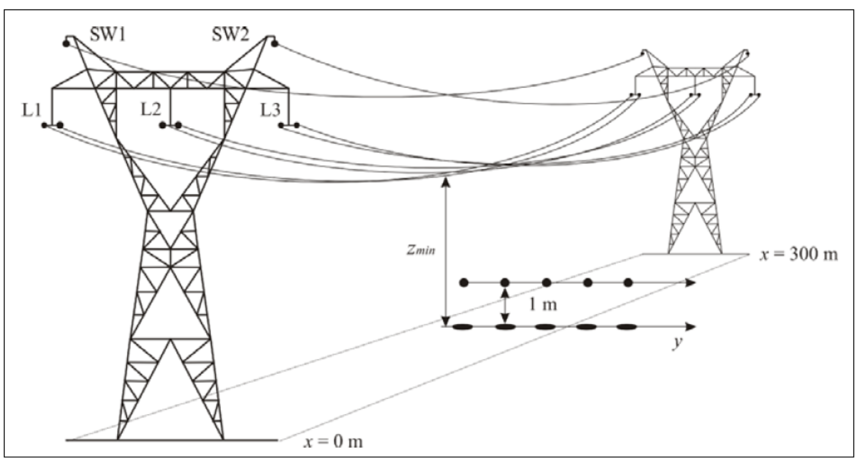

Figure 7: Setup of the numerical example in [2]. To avoid confusion, please notice that the orientation of the x, y and z-axis in my paper is different than the orientation of the x, y and z-axis in [2].

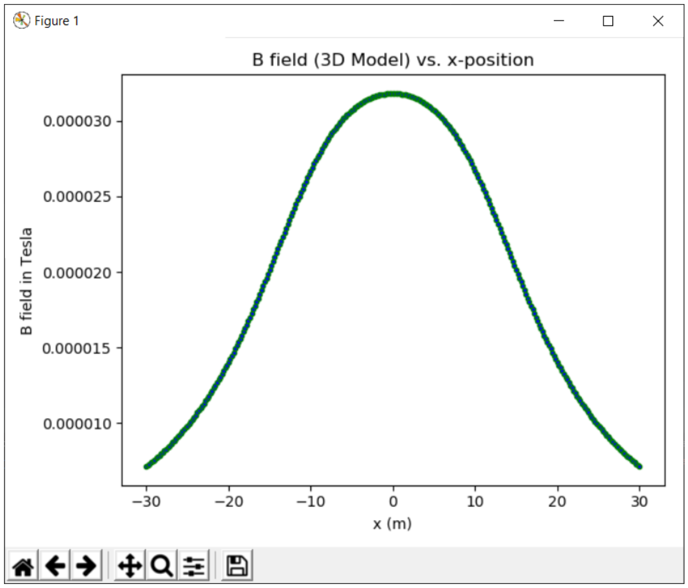

Figure 8: My answer to the numerical example in [2].

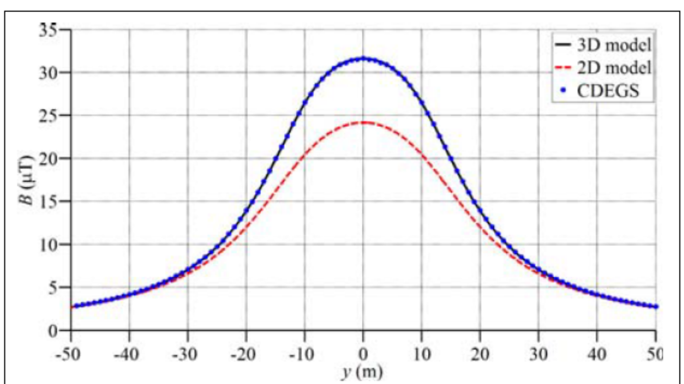

Figure 8. Effective values of the total magnetic flux density.

Figure 9: The answer shown in [2]. To avoid confusion, please notice that the orientation of the x, y and z-axis in my paper is different than the orientation of the x, y and z-axis in [2].

7 Conclusion

In this section, I would like to talk about two topics. The first topic is the strength of the magnetic field beneath a 3-phase transmission circuit versus the strength of the magnetic field emitted by common household appliances. The second topic is my thoughts about the power of high school mathematics.

As shown in Figure 8, the maximum magnetic field strength in the numerical example is approximately $|B| = 32\mu$T. However, this maximum value only occurs when the person is standing directly underneath Conductor 2 at coordinate (x_p, y_p, z_p) = (0, 1, 0). People usually do not live directly underneath a transmission circuit. Based on Figure 8, we see that if a person was to stand at coordinate (x_p, y_p, z_p) = (30, 1, 0) or (-30, 1, 0), i.e., 20 m away from the closest conductor, then the magnetic field strength drops dramatically to approximately $|B| = 2\mu$T. As comparison, here are maximally-measured magnetic field strengths at various distances away from some common electrical appliances, measured in μT; see [5]:

Appliance	3 cm	30 cm	1 m
Hair dryer	2000	7	0.03
Electric shaver	1500	9	0.03
Drill	800	3.5	0.2
Vacuum cleaner	800	20	2
Fluorescent light	400	2	0.25
Microwave oven	200	8	0.6
Portable radio	56	1	0.01
Electric oven	50	0.5	0.04
Washing machine	50	3	0.15
Iron	30	0.3	0.03
Dishwasher	20	3	0.3
Computer	30	0.01	0
Refrigerator	1.7	0.25	0.01
Television	50	2	0.15

To determine the typical magnetic field strength that a person is exposed to, I will only look at items that are turned on constantly or for long durations. These items include the fluorescent light, computer, refrigerator and old tube television sets. However, I will eliminate the refrigerator because people do not usually stand for long durations near their refrigerators. So, there are only three items of concern. I estimate that people are usually 1 m away from a fluorescent light, so the magnetic field strength will be 0.25 μT. I estimate that people are usually sitting 30 cm away from the edge of their laptop computer when in use, so the magnetic field strength will be 0.01 μT. I estimate that people are usually more than 1 m away from a television when in use, so the magnetic field strength will be much less than 0.15 μT. So, it seems that living at the edge of a transmission corridor could potentially expose you to a greater magnetic field strength than normal, especially if you consider the fact that a transmission circuit is energized every second of every day. However, if you regularly use a hair dryer, electric shaver, or vacuum cleaner, then the added magnetic field strength from a transmission circuit is likely negligible.

In this paper, I have presented an easy mathematical process to calculate the magnetic field strength beneath a 3-phase transmission circuit. I have explained my process using only algebra and trigonometry. My process is similar to the process published in [2] by three university professors. However, my process is explained using simple mathematics. To test the accuracy of my process, I repeated a numerical example shown in [2] and achieved very similar results. This proves that my process and my Python code is accurate. My paper shows that it is possible for high school students to use high school level mathematics to understand and replicate university-level work.

A Python Code

```
from mpmath import *
import numpy as np
import matplotlib.pyplot as plt

I1 = 2*960*(cos(radians(-30))+j*sin(radians(-30)))
I2 = 2*960*(cos(radians(-150))+j*sin(radians(-150)))
I3 = 2*960*(cos(radians(90))+j*sin(radians(90)))

L = 300
z = -L/2
x1 = -10
x2 = 0
x3 = 10
yinterest = 1
zinterest = 0
constant = 1*10**-7
pieces = 100
deltaz = L/pieces

curvature = (L/2)/(np.arccosh(20/12.5))

for count in range(-150, 151):
   xinterest = count*0.2
   Bx = 0
   By = 0

   for count1 in range(1,pieces+1):
      y1 = 12.5 * np.cosh(z / curvature)
```

```
        deltaBx = -1*(yinterest-y1)*constant*I1*deltaz/(((xinterest - x1)**2 + (yinterest - y1)**2 + (zinterest - z)**2)**1.5)
        deltaBy = (xinterest-x1)*constant*I1*deltaz/(((xinterest - x1)**2 + (yinterest - y1)**2 + (zinterest - z)**2)**1.5)
        Bx = Bx + deltaBx
        By = By + deltaBy
        z = z + deltaz

    z = -L/2

    for count2 in range(1,pieces+1):
        y2 = 12.5 * np.cosh(z / curvature)
        deltaBx = -1*(yinterest-y2)*constant*I2*deltaz/(((xinterest - x2)**2 + (yinterest - y2)**2 + (zinterest - z)**2)**1.5)
        deltaBy = (xinterest-x2)*constant*I2*deltaz/(((xinterest - x2)**2 + (yinterest - y2)**2 + (zinterest - z)**2)**1.5)
        Bx = Bx + deltaBx
        By = By + deltaBy
        z = z + deltaz

    z = -L / 2

    for count3 in range(1,pieces+1):
        y3 = 12.5 * np.cosh(z / curvature)
        deltaBx = -1*(yinterest-y3)*constant*I3*deltaz/(((xinterest - x3)**2 + (yinterest - y3)**2 + (zinterest - z)**2)**1.5)
        deltaBy = (xinterest-x3)*constant*I3*deltaz/(((xinterest - x3)**2 + (yinterest - y3)**2 + (zinterest - z)**2)**1.5)
        Bx = Bx + deltaBx
        By = By + deltaBy
        z = z + deltaz

    z = -L/2

    B = (Bx.real ** 2 + Bx.imag ** 2 + By.real ** 2 + By.imag ** 2) ** 0.5
    plt.subplot(1, 1, 1)
    plt.plot(xinterest, B, color='green', linestyle='dashed', linewidth=1, marker='o', markerfacecolor='blue', markersize=3)

plt.subplot(1, 1, 1)
plt.xlabel('x␣(m)')
plt.ylabel('B␣field␣in␣Tesla')
plt.title('B␣field␣(3D␣Model)␣vs.␣x-position')
plt.tight_layout()
plt.show()
```

References

[1] R. Stam, Comparison of international policies on electromagnetic fields (power frequency and radio frequency fields), National Institute for Public Health and the Environment, RIVN, the Netherlands, pp. 1-20, 2018.

[2] T. Modric, S. Vujevic and D. Lovrić, 3D Computation of the Power Lines Magnetic Field, *Progress In Electromagnetics Research M* **41** (2015), 1–9.

[3] S. Vujevic, P. Sarajcev and A. Botica, Computation of the overhead power line electromagnetic field, *16th International Conference on Software, Telecommunications and Computer Networks, Split, Croatia* (2008), 27–31.

[4] S. Vujevic, P. Sarajcev and D. Lovrić, Computation of the Power Line Electric and Magnetic Fields, *17th Telecommunications Forum TELFOR 2009, Belgrade, Serbia* (2009), 875-878.

[5] H. Heller (ed.), Schutz vor niederfrequenten elektrischen und magnetischen Feldern der Energieversorgung und -anwendung, Berichte der Strahlenschutzkommission, Article 7, 1997. Table replicated here:
https://www.bfs.de/EN/topics/emf/lff/application/household-electrical/household-electrical.html,
last accessed on 2022-03-10.