Parabola Volume 58, Issue 2 (2022)

Atmospheric pressure waves from the Hunga Tonga-Hunga Haʻapai volcanic eruption

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1 Introduction

On 15 January, 2022 the undersea Hunga Tonga-Hunga Haʻapai volcano located in the Kingdom of Tonga erupted. It was the largest volcanic eruption since that of Mount Pinatubo in 1991, and created a tsunami that affected the entire Pacific basin. The eruption and tsunami caused significant damage to the islands of Tonga.

Figure 1: Satellite image of the Hunga Tonga-Hunga Haʻapai eruption (left panel), and a photograph of damage to Nomuka island (right panel).
Image credits: NASA Earth Observatory and Malau Media [1, 7].

In addition to the tsunami waves, an atmospheric pressure wave was generated by the eruption. This pressure wave was recorded globally, and circled the Earth at least three times. The wave is easily seen in Figure 2, which shows the pressure record from Honolulu, Hawaiʻi for the five days following the eruption. Data are from the US Automated Surface Observing System, and were obtained via the Iowa Environmental Mesonet's web interface [6].

In this short article, we show how maths can be used to analyse some of the pressure wave's properties.

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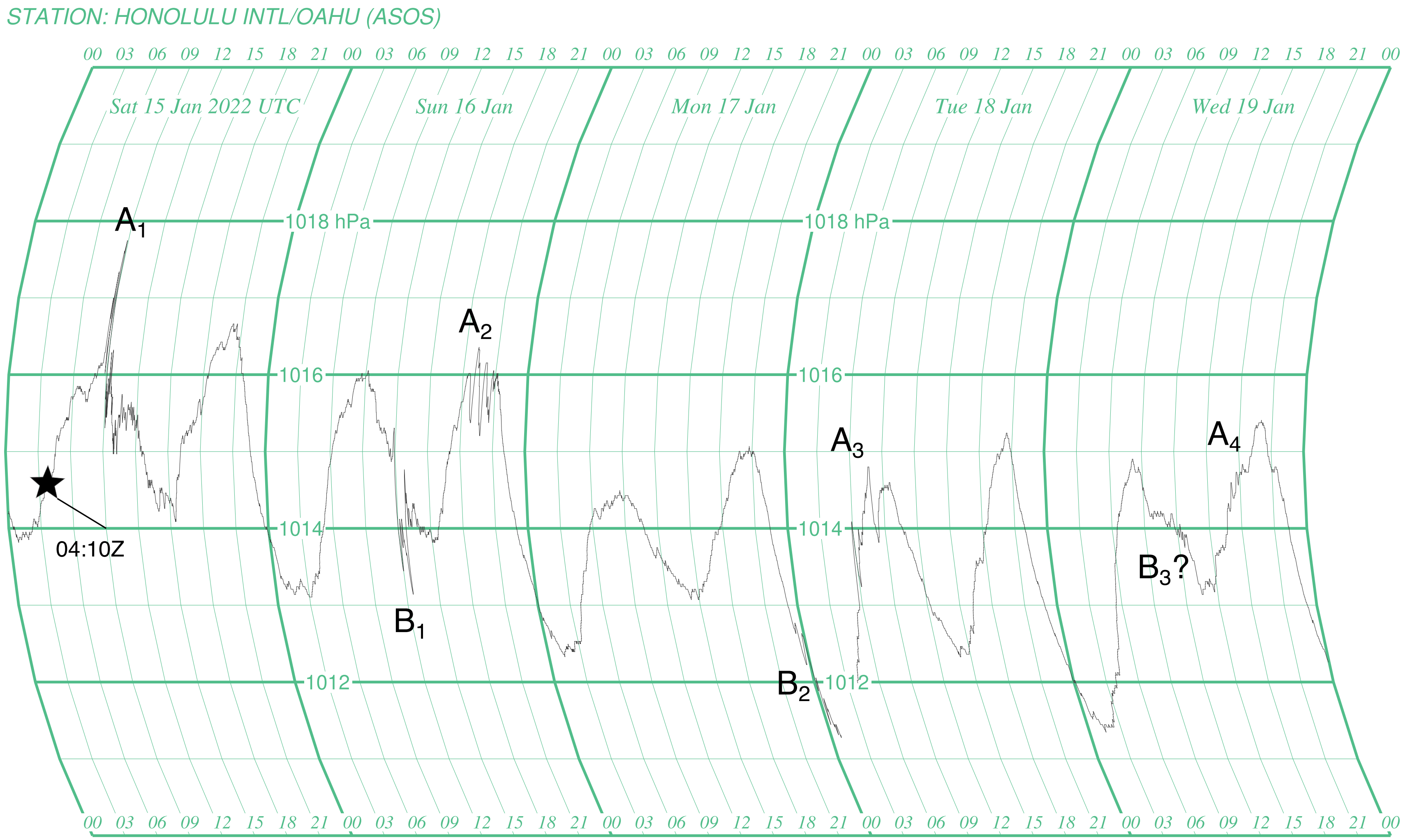

Figure 2: Barograph showing the atmospheric pressure measured at Honolulu International Airport from 15–19 January, 2022. The Hunga Tonga-Hunga Haʻapai volcano erupted at 04:10Z on 15 January. Waves associated with the volcanic eruption are labelled $A_1, \ldots, A_4$ and B_1, B_2, B_3. All times are in Coordinated Universal Time (UTC).

2 Atmospheric pressure waves

Large volcanic eruptions can generate atmospheric pressure waves of global extent. For example, the eruptions of Krakatau in 1883 and Mount Pinatubo in 1991 generated atmospheric pressure waves which were measured around the world [13, 17]. There are numerous other reports of pressure waves generated by volcanoes in the literature.

Other explosive events can also create globally observed pressure waves. For example, the pressure wave from the Soviet Tsar Bomba nuclear test in 1961 was recorded three times at Wellington, New Zealand [4].

Figure 3: From left to right, Mount Krakatau, Mount Pinatubo and the Tsar Bomba nuclear test all produced global atmospheric pressure waves.
Image credits: Parker and Coward, D. Harlow and ROSATOM [5, 14, 16].

In Figure 2, we can clearly see the pressure waves from the Hunga Tonga-Hunga Haʻapai eruption arriving at Honolulu. These are labelled as $\mathrm{A}_1, \ldots, \mathrm{A}_4$ and $\mathrm{B}_1, \mathrm{B}_2, \mathrm{B}_3$ on the barograph. The following sections analyse some of the properties of these waves.

Speed of pressure waves

A fundamental property of any wave is its speed. To compute the speed of a wave, we need to measure the time taken by the wave to travel a given distance.

$$v = \frac{\ell}{\delta t}, \tag{1}$$

where v is the wave's speed, and δt is the time taken for it to traverse the distance ℓ.

We can compute how long the first wave, A_1, took to travel from Hunga Tonga-Hunga Haʻapai to Honolulu from the Honolulu barograph. It helps to zoom in on the barograph to get an accurate timing. Figure 4 shows the pressure record at Honolulu for January 15, 2022. The volcanic eruption was at approximately 04:10Z, and the first wave is observed at Honolulu at approximately 08:40Z, making the travel time, δt, 4 hours and 30 minutes.

We now compute the distance between Hunga Tonga-Hunga Haʻapai and Honolulu. The pressure wave will travel on the shortest path between the two locations. We assume the Earth is spherical, with a radius of $6371\ \mathrm{km}$[2]. The shortest path between two

[2]In reality, the Earth is an oblate spheroid but the difference from a sphere is minor.

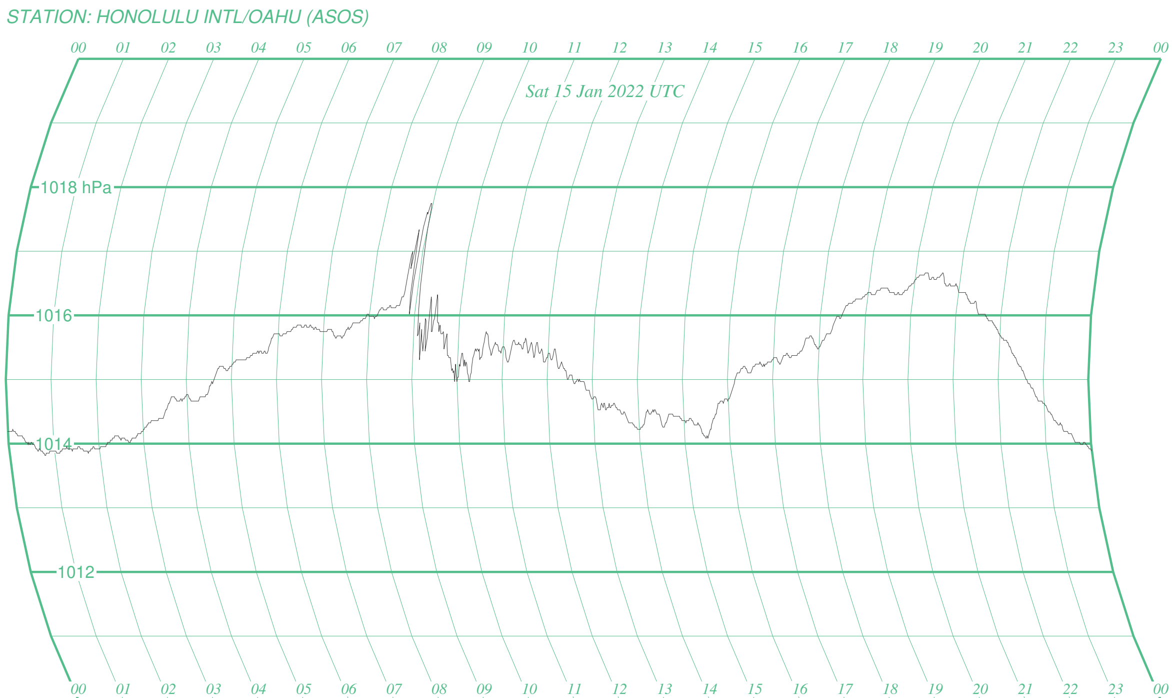

Figure 4: Honolulu barograph for January 15, 2022.

points on a sphere is the great circle arc between them, as illustrated in Figure 5.

It is customary to reference locations on the Earth's surface using geographic coordinates, by specifying a longitude and latitude. In this article, longitudes are represented by the symbol λ, and latitudes by φ. Latitude ranges from $-90°$ at the South Pole to $90°$ at the North Pole, and is $0°$ along the Equator. Longitude is defined to be $0°$ at the Prime Meridian, which passes through a point very near the Royal Observatory in Greenwich, London[3]. Positive longitudes lie to the east of the Prime Meridian, and negative values to the west.

However, for our purposes of computing the length of a great circle arc, it is more convenient to specify locations in Cartesian coordinates. We define the Z axis of the Cartesian coordinate system to run through the Earth's North and South poles. The plane defined by the Z and Y axes passes through the Prime Meridian, and the plane defined by the X and Y axes intersects the Equator. This is illustrated in Figure 6.

If the longitudes of the two points of interest on the Earth's surface are (λ_1, φ_1) and (λ_2, φ_2), respectively, then the corresponding Cartesian coordinates are:

$$(x_1, y_1, z_1) = (R_e \sin\lambda_1 \cos\varphi_1, R_e \cos\lambda_1 \cos\varphi_1, R_e \sin\varphi_1), \tag{2}$$
$$(x_2, y_2, z_2) = (R_e \sin\lambda_2 \cos\varphi_2, R_e \cos\lambda_2 \cos\varphi_2, R_e \sin\varphi_2)\,. \tag{3}$$

[3]Until 1984, the Prime Meridian passed through the Royal Observatory. Following a redefinition of the Prime Meridian in 1984, it now lies about $102\,\mathrm{m}$ away from the observatory.

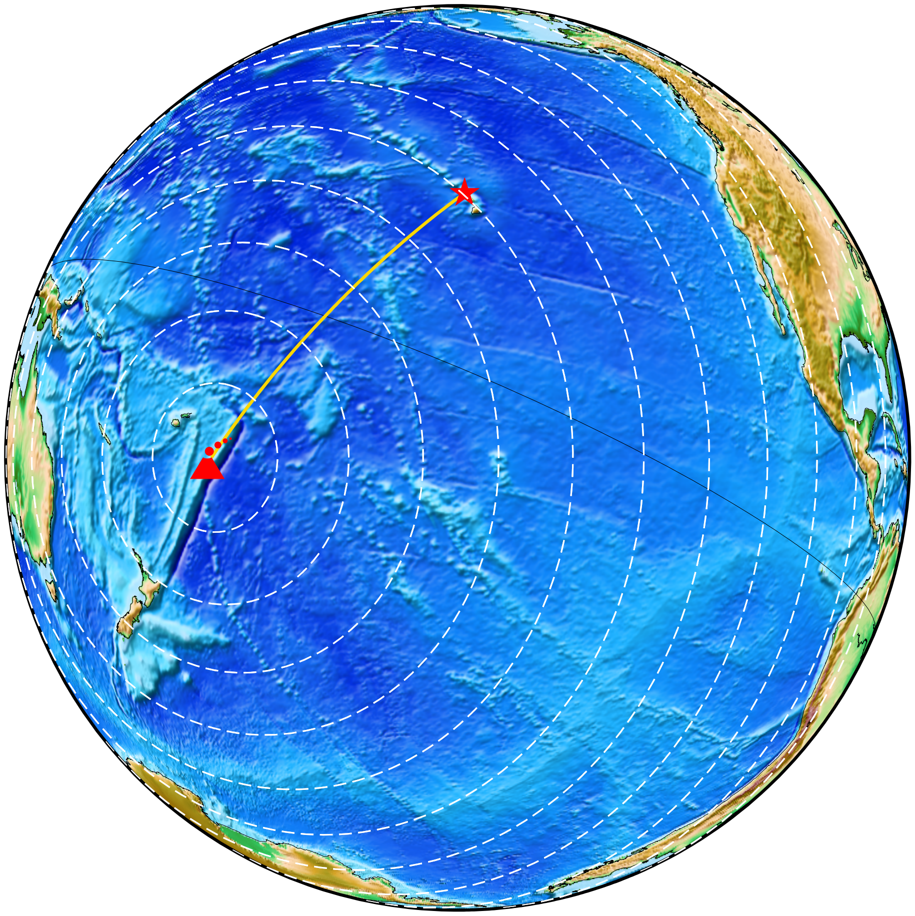

Figure 5: Great circle path (gold line) between Hunga Tonga-Hunga-Haʻapai (volcano symbol) and Honolulu (star). The white dashed circles indicate the propagating pressure wavefront, in $1000\,\mathrm{km}$ increments. The black line is the Equator.

The straight line distance, d, between (x_1, y_1, z_1) and (x_2, y_2, z_2) is computed using Pythagoras' Theorem in three dimensions.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\,. \tag{4}$$

This distance, d, is sometimes referred to as the tunnel distance, because it would be the length of a tunnel bored in a straight line between the two points on the Earth's surface.

We can now compute the great circle arc length, shown as ℓ in Figure 6. Consider the angle α formed between the two points on the Earth's surface and the centre of the Earth:

$$\sin\left(\frac{\alpha}{2}\right) = \frac{d}{2R_e}\,. \tag{5}$$

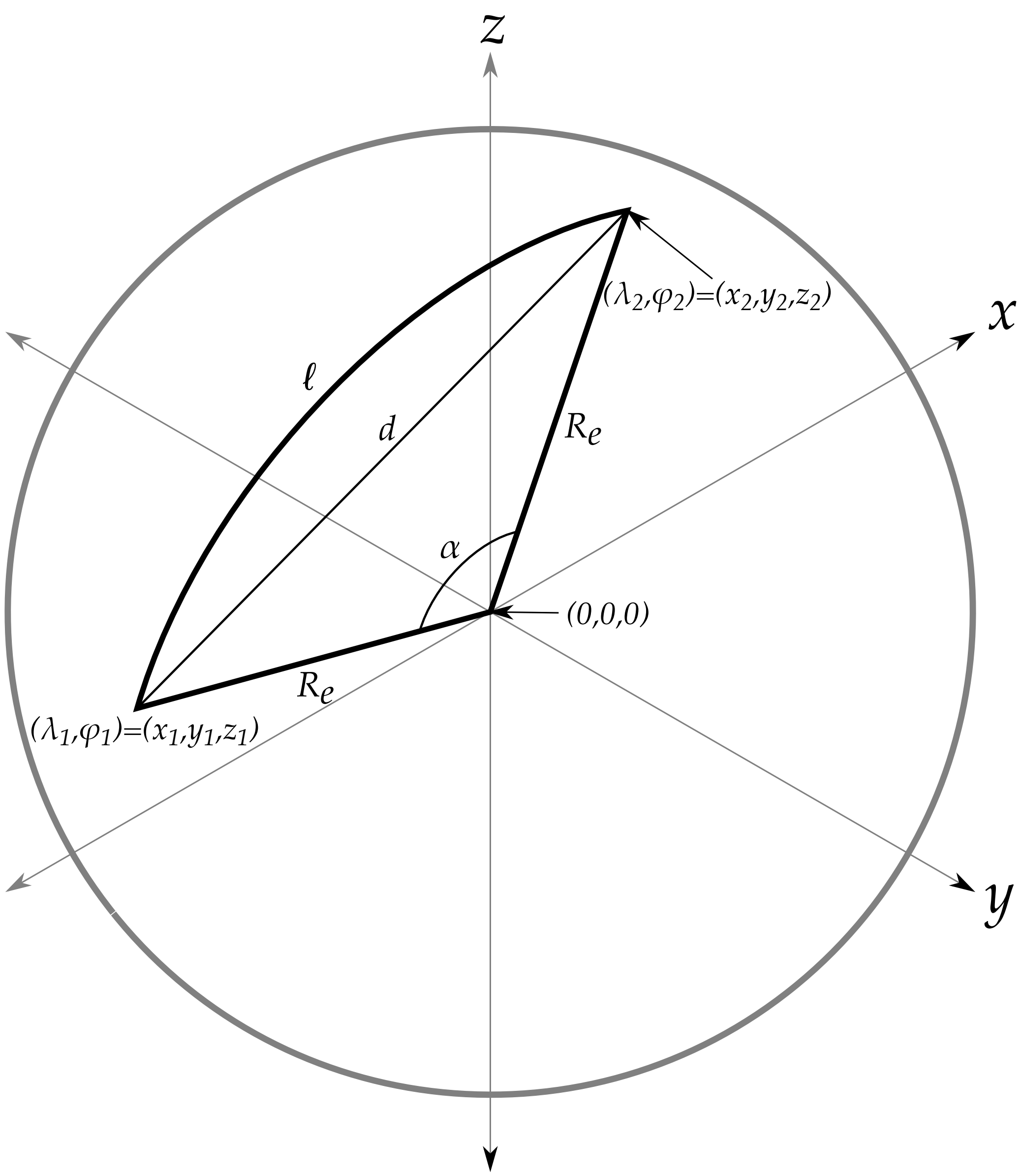

Figure 6: Locations of points in polar and Cartesian coordinates.

Therefore,

$$\alpha = 2\arcsin\left(\frac{d}{2R_e}\right) . \tag{6}$$

If we have computed α in radians, then the great circle arc length is just

$$\ell = \alpha R_e . \tag{7}$$

Starting with the locations of the two points on the Earth's surface, (λ_1, φ_1), and (λ_2, φ_2), we can use Equations 2–7 to compute the great circle arc length. Without further ado, we substitute the locations of Hunga Tonga-Hunga Haʻapai, Honolulu and the radius of the Earth into the equations:

$$\begin{aligned}(\lambda_1, \varphi_1) &= (-175.3888°, -20.545\,25°) ,\\ (\lambda_2, \varphi_2) &= (-157.924\,47°, 21.317\,61°) ,\\ R_e &= 6371\,\mathrm{km} .\end{aligned}$$

These values give the great circle arc length between the two locations of $5027\,\mathrm{km}$[4].

Some readers may balk at using several equations to compute the distance between two points on the Earth. Surely, there must be a more elegant way? And they'd be correct. The following formula is an elegant simplification of Equations 2–7 [18]:

$$\ell = R_e \arccos(\sin\varphi_1 \sin\varphi_2 + \cos\varphi_1 \cos\varphi_2 \cos(\lambda_2 - \lambda_1)) . \tag{8}$$

The proof of this formula is left as an interesting exercise for the reader.

We now have the information required to compute the speed of the pressure wave using Equation 1. The speed of the wave is approximately $1117\,\mathrm{km\,h^{-1}}$. This is about $90\,\%$ the speed of sound in air. The sound of the eruption was heard as far away as Alaska; no doubt the audible sound and the pressure wave are related [12].

Subsequent waves

So far the properties of the first wave to reach Honolulu (A_1) have been investigated. However, a number of subsequent waves are apparent in Figure 2.

We first look at waves $\mathrm{A}_2, \mathrm{A}_3, \mathrm{A}_4$. These waves arrive at approximately 35 hour intervals; the exact timing is difficult to determine because each wave arrives over about a three hour period. If we assume these waves are travelling at the same speed as the A_1 wave, then it is apparent they have travelled over $39\,000\,\mathrm{km}$. This is close to the circumference of the Earth. This suggests that waves $\mathrm{A}_2, \mathrm{A}_3, \mathrm{A}_4$ are just the original wave circumnavigating the world a further three times.

The explanation for waves $\mathrm{B}_1, \mathrm{B}_2, \mathrm{B}_3$ is similar to the A waves; however, in these cases it is the pressure wave going in the opposite direction around the world to the A wave.

[4]One could have used Google Maps to measure the distance between Hunga Tonga-Hunga Haʻapai and Honolulu but there is no fun in doing it this way.

Atmospheric tides

We have investigated the rapid fluctuation in pressure caused by the passage of the volcanic pressure wave. Readers may be wondering what the large recurring pressure variations with a period of 12 hours are? These are a commonly observed phenomenon in the tropics called the atmospheric tide. The atmospheric tide is unrelated to the volcanic eruption. Atmospheric tides are not often seen outside the tropics because they get swamped by much larger pressure changes associated with the passage of mid-latitude weather systems.

The name atmospheric tide is perhaps confusing to anyone familiar with the ocean tides. Oceanic tides are caused by the gravitational force from the Moon and Sun [9]. One cycle of ocean tides is 24 hours and 50 minutes, though in this cycle we usually get two high tides. However, if we study the atmospheric tide evident in Figure 2, then we notice two cycles in exactly 24 hours. This exact diurnal cycle suggests atmospheric tides are predominantly driven by solar heating (which has a 24 hour cycle) rather than gravitational forces, although a weaker lunar tide can sometimes be detected in the atmospheric tide.

A detailed discussion of the mechanisms responsible for atmospheric tides is beyond the scope of this article. Interested readers may find the introduction in Covey *et al.* useful [3].

3 Summary

An atmospheric pressure wave caused by the Hunga Tonga-Hunga‘apai volcanic eruption circled the Earth at least three times and was observed in the barograph record from Honolulu. We showed that this wave was travelling at almost the speed of sound in air. By the time the wave circled the Earth three times (A_4) it was quite weak. However, with careful analysis of barograph records from other stations around the world it may be possible to find a fourth circumnavigation.

A common question which students may have is what jobs can they do with maths? All of the geophysical sciences, such as meteorology, vulcanology, seismology and oceanography require a sound background in applied mathematics. Hopefully, this article has provided a glimpse into how maths can have very interesting and useful applications in the real world.

Acknowledgements

I would like to thank Thomas Britz, Sin Keong Tong and several colleagues from the Bureau of Meteorology for discussions which helped improve this article.

Addendum

This article was written shortly after the Hunga Tonga-Hunga Haʻapai eruption. In the following months, there has been a large amount of research into the volcanic eruption and tsunami.

The eruption's magnitude has been confirmed to be comparable to that of Krakatau in 1883 [15]. The atmospheric shock-wave was recorded by a large number of geophysical instruments around the world; this is described in [8].

The tsunami associated with the eruption affected the whole Pacific basin. A small tsunami was also recorded in the Caribbean at the same time as the passage of the shock-wave [11]. Normally, a tsunami in the Pacific Ocean would be blocked by the Central American land mass from reaching the Caribbean. This suggests that the tsunami was at least partially caused by the shock-wave, a phenomenon known as a meteo-tsunami [10].

The people of Tonga are gradually recovering from the eruption and tsunami but the effects will be felt for many years to come. A recent Australian Broadcasting Corporation news article describing the impact six months afterwards may be of interest to readers [2].

References

[1] S. Bates and M. Carlowicz, *Tonga Volcano Plume Reached the Mesosphere*, https://earthobservatory.nasa.gov/images/149474/tonga-volcano-plume-reached-the-mesosphere, last accessed on 2022-03-13.

[2] A. Burgess and A. Jacobson, *Six months after volcanic eruption and tsunami, Tongans who lost everything have plans for their future*, https://www.abc.net.au/news/2022-07-15/tonga-volcano-tsunami-six-months-recovery-resettlement-mango/101216258, last accessed on 2022-07-15.

[3] C. Covey, A. Dai, D. Marsh and R. S. Lindzen, The surface-pressure signature of atmospheric tides in modern climate models, *Journal of the Atmospheric Sciences* **68** **(3)** (2011), 495–514.

[4] E. Farkas, Transit of pressure waves through New Zealand from the Soviet 50 megaton bomb explosion, *Nature* **193** (1962), 765–766.

[5] D. Harlow, *The June 12, 1991 eruption column from Mount Pinatubo taken from the east side of Clark Air Base*, https://www.usgs.gov/media/images/june-12-1991-eruption-column-mount-pinatubo-taken-th, last accessed on 2022-03-13.

[6] Iowa State University, *Iowa Environmental Mesonet*, https://mesonet.agron.iastate.edu/request/asos/1min.phtml, last accessed on 2022-03-13.

[7] Malau Media, *Malau Media*, https://www.facebook.com/malaumedia, last accessed on 2022-03-13.

[8] R.S. Matoza and 76 others, Atmospheric waves and global seismoacoustic observations of the January 2022 Hunga eruption, Tonga, *Science* **377** (**6601**) (2022), 95–100.

[9] National Oceanic and Atmospheric Administration, *What Causes Tides?*, https://scijinks.gov/tides/, last accessed on 2022-03-13.

[10] National Oceanic and Atmospheric Administration, *What is a meteotsunami*, https://oceanservice.noaa.gov/facts/meteotsunami.html, last accessed on 2022-07-15.

[11] National Oceanic and Atmospheric Administration, *Ripple effect: What the Tonga eruption could mean for tsunami research*, https://www.noaa.gov/news/ripple-effect-what-tonga-eruption-could-mean-for-tsunami-research, last accessed on 2022-07-14.

[12] M. Nelson and R. McChesney, *The eruption near Tonga was so powerful you could hear it in Alaska*, https://www.alaskapublic.org/2022/01/17/the-eruption-near-tonga-was-so-powerful-you-could-hear-it-in-alaska, last accessed on 2022-03-13.

[13] G. Pararas-Carayannis, Near and far-field effects of tsunamis generated by the paroxysmal eruptions, explosions, caldera collapses and massive slope failures of the Krakatau volcano in Indonesia on August 26–27, 1883, *The International Journal of the Tsunami Society* **21** (**4**) (2003), 191–201.

[14] Parker and Coward, View of Krakatoa during the Earlier Stage of the Eruption from a Photograph taken on Sunday the 27^{th} of May, 1883, *The eruption of Krakatoa, and subsequent phenomena. Report of the Krakatoa Committee of the Royal Society*, (1888), https://en.wikipedia.org/wiki/File:Krakatoa_eruption_lithograph.jpg, last accessed on 2022-03-13.

[15] P. Poli, N..M. Shapiro, Rapid Characterization of Large Volcanic Eruptions: Measuring the Impulse of the Hunga Tonga Haʻapai Explosion From Teleseismic Waves, *Geophysical Research Letters* **49** (**8**) (2022).

[16] The State Atomic Energy Corporation ROSATOM, *Испытание чистой водородной бомбы мощностью 50 млн тонн*, 2020.

[17] M. Tahira, M. Nomura, Y. Sawada and K. Kamo, Infrasonic and Acoustic-Gravity Waves Generated by the Mount Pinatubo Eruption of June 15, 1991, *Fire and Mud: Eruptions and Lahars of Mount Pinatubo, Philippines*, (1997), https://pubs.usgs.gov/pinatubo/tahira/, last accessed on 2022-03-13.

[18] Wikipedia, *Great-circle distance*, https://en.wikipedia.org/wiki/Great-circle_distance, last accessed on 2022-03-13.