# **Optimizing team performance in tug of war: a gradient descent approach**

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#### **1 Introduction**

Mathematical modelling of sports competitions can be of great interest not only to mathematicians but also to athletes, coaches and students [\[2,](#page-10-0) [4,](#page-10-1) [5,](#page-10-2) [12,](#page-11-0) [13,](#page-11-1) [17,](#page-11-2) [22\]](#page-12-0). Optimization theory is one such approach that can be used to model and analyse sports competitions.

In competitive environments, such as a sports tournament, maximizing the benefit of one group depends not only on its own strengths but also on the weaknesses of its competitors. This highlights the importance of considering both individual and team performances in developing effective strategies for winning. By optimizing the performance of individual players and the team as a whole, it is possible to identify the most effective strategies for winning. This approach has been applied to a variety of sports, including soccer, basketball and baseball, with promising results [\[2,](#page-10-0) [4,](#page-10-1) [5,](#page-10-2) [12,](#page-11-0) [13,](#page-11-1) [17,](#page-11-2) [22\]](#page-12-0). However, to the best of our knowledge, optimization theory has not been applied extensively to the sport of tug of war.

Friction plays a crucial role in the game of tug of war, influencing the ability of teams to exert force and gain traction. The interaction between the players' feet and the surface they are standing on determines the level of friction and ultimately affects their performance. The type of shoes worn by the players and the characteristics of the playing surface, such as its texture and grip, directly impact the frictional forces involved. Moreover, the angles of the ankles, knees and waists of the players contribute to maximizing the pulling force. These body mechanics principles highlight the importance of optimizing the players' positions and movements to enhance their ability to generate and transmit force effectively through the rope.

The game of tug of war involves not only individual efforts but also the collective synergy of the team. Synergy refers to the concept that the combined effort of two individuals working together can achieve more than the sum of their individual efforts. The coordination and synchronization of pulling forces among team members are essential for maximizing the overall strength exerted on the rope. However, as the game progresses, the players experience fatigue, which impacts their performance. Fatigue can arise from a variety of physiological factors, including the accumulation of lactic acid, depletion of energy stores and muscle fatigue. Understanding the interplay between synergy and fatigue is crucial for devising effective strategies in tug of war. By

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developing a mathematical model that incorporates these factors, it becomes possible to analyze the optimal allocation of energy and effort among team members, considering the balance between maintaining synergy and mitigating the effects of fatigue.

Therefore, in this paper, we aim to fill this gap by using optimization theory to determine the factors that govern the triumph of a tug of war team. Specifically, we will develop a mathematical model that takes into account the strengths of individual players, the synergy between players and the effect of fatigue on the outcome of the game. In the following section, our attention shifts towards delving into the intricate biomechanics inherent in the game of tug of war.

### **2 Biomechanics and tug of war**

The game of tug of war measures the strength of two opposing teams as they pull a rope from opposite ends. The objective of the game is to pull the rope to a predetermined distance of 4 meters on one side, and the team that achieves this goal is declared the winner. The aim of this paper is to identify the crucial physical factors that can significantly enhance the chances of winning in a game of tug of war. As an example, one of the most critical factors that affect the outcome of the game is friction. In this regard, this paper summarizes some of the essential factors that can influence the likelihood of a team's success in the game.

In a game of tug of war, a team's chances of winning increase when they are able to exert more force on the ground and increase the friction between their feet and the surface they are standing on. Heavier individuals are often selected for such contests. The type of shoes and the surface of the floor are also crucial in determining the level of friction [\[26\]](#page-12-1).

The strength of the hands and the friction between the hands and the rope are important factors in determining the outcome of the game. Individuals with stronger hands are better able to press the rope and exert force, thus enhancing the team's chances of winning.

The angles of the ankles, knees and waists are crucial in maximizing the pulling force. The relationship between the angles of the ankle and waist is positive, while that between the angle of the knee and the angles of the ankle and waist is negative [\[10\]](#page-11-3).

Holding the rope higher during the contest is associated with lower pulling force [\[10\]](#page-11-3).

The inclination of the body negatively impacts pulling force [\[10\]](#page-11-3).

Wearing a waist belt can be helpful in tug of war [\[23\]](#page-12-2).

Timing is crucial in tug of war as even small delays among team members in exerting their force can lead to a significant loss in total force [\[24\]](#page-12-3).

Maintaining the rope horizontally is important, and having team members of similar height can help reduce force decomposition in the y-direction.

The ability to sustain physical strength and endurance is an important factor in winning.

We assume that both teams begin the game with similar conditions, including members of roughly equal weight, strong hands, similar heights, equal physical strength and similar techniques. Under these conditions, it might seem that neither team has an advantage, but this paper predicts that the outcome depends on the strategies employed by each player in force exertion.

## **3 A mathematical approach to modelling synergy, fatigue and utility**

The tug of war game is a classic contest of strength and strategy between two teams. In this game, each team tries to pull the rope towards their own side, with the goal of bringing their opponent's flag past the centre line. To win, a team must successfully pull their opponent's flag all the way to their own side. For simplicity, we consider a two-player model of tug of war in this paper.

But what if we could optimize our strategy for winning the tug of war game? That's where the concept of utility comes in. Utility is a measure of the usefulness or value of a particular action or strategy, and in the context of the tug of war game, it can help us determine the best way to pull the rope.

To calculate utility, we can use a mathematical model that takes into account two important factors: synergy and fatigue. Synergy is the concept that two individuals working together can achieve more than they could separately, while fatigue is the idea that the more we work, the less effective we become. By incorporating these concepts into our model, we can determine the optimal way to pull the rope to win the game.

<span id="page-2-0"></span>
$$
U(F_1, F_2) = (F_1 + F_2 + S(F_1, F_2)) \times F(F_1, F_2)
$$
\n(1)

in which U,  $F_1$ ,  $F_2$ , S and F are the utility function, the force exerted by Player 1, the force exerted by Player 2, the synergy term and the fatigue respectively. In the model,  $S = kF_1F_2$  in which k is the synergy constant, which represents the degree to which two individuals working together can achieve more than they could separately. Also,  $F = e^{-c(F_1 + F_2)}$  in which c is the fatigue constant, which represents the rate at which we become less effective as we work harder. Note that Equation [\(1\)](#page-2-0) is a normalized non-dimensional equation.

One of the main causes of muscle fatigue is the accumulation of lactic acid, which is a waste product produced during the glycolytic pathway. When there is an excessive accumulation of lactic acid, it can inhibit the activity of glycolytic enzymes and result in muscle fatigue [\[21\]](#page-12-4). In the context of sports science, fatigue is a complex and multifaceted phenomenon that can have a significant impact on athletic performance. However, defining and measuring fatigue can be challenging due to variations in terminology and understanding across different domains. Researchers have used terms

such as endurance, exhaustion and fatigue in different ways, which can complicate efforts to compare and generalize findings.

In the case of muscle fatigue, it is typically defined as the point at which a muscle is no longer able to sustain its force or power output. This can occur when the muscle is unable to generate enough energy in the form of ATP to meet the demands of the activity.

To model fatigue in the context of a tug of war game, it is assumed that the strength of each player will decrease exponentially over time as a result of the ongoing physical exertion. This assumption is supported by previous research in the field [\[11,](#page-11-4) [15,](#page-11-5) [9\]](#page-11-6), which has demonstrated that fatigue can have a significant impact on performance in high-intensity activities like tug of war. By incorporating this fatigue factor into the model, we can gain a more accurate understanding of how different factors influence the outcome of the game. The Appendix includes a brief review of skeletal muscle characteristics for readers who are interested in further details.

To optimize our strategy, we will use gradient descent, a popular optimization algorithm in machine learning [\[18\]](#page-12-5). We will iteratively update our pulls in the direction that maximizes the utility function, with the goal of finding the optimal pulls to win the game.

Our model has several hyperparameters that we can adjust to fine-tune our strategy, including the learning rate (the step size of our updates) and the maximum number of iterations (how many times we will update our pulls). In the context of the simulation, an "iteration" refers to a single round of the game in which each player makes a move based on their current strategy and the state of the game. However, in a real game, iterations could be thought of as discrete time steps in which each team makes a move and the game state evolves accordingly. The learning rate determines the speed at which the players learn from their previous experiences. A higher learning rate may lead to faster adaptation to the game, while a lower learning rate may result in a more gradual learning process. By experimenting with different hyperparameters, we can find the optimal strategy to win the tug of war game.

The initial values of  $F_1$  and  $F_2$  for the players of the first team are set to 50, while for the second team, it is assumed to be 70 and 30, respectively. Even though the sum of the forces of each team is equal to the other team, the different distribution of forces between the teams is expected to have a significant impact on the outcome of the game. The model was numerically simulated and the results are shown in Figures [1\(](#page-4-0)a) to [1\(](#page-4-0)d). The constants  $k, c$ , and learning rate were set to 0.1, 0.01 and 0.01 respectively. As shown in the figure, the team with an equal distribution of strength among its players (Team 1) has a higher probability of winning the game compared to the team with nonuniform force distribution (Team 2). This indicates that if one player puts less effort, they act as a hindrance to their teammate, wasting their partner's power.



<span id="page-4-0"></span>Figure 1: Simulation results of a two-player model of tug of war game.

To provide insights into the robustness of the model and the impact of different parameters on the game outcome, we perform a sensitivity analysis on the input parameters of the model (synergy constant  $k$ , fatigue constant  $c$  and learning rate). This could involve testing the impact of varying these parameters on the optimal forces and utility values obtained by the teams.

The results (Figure [2\)](#page-5-0) show that in all simulations, Team 1 has a higher chance of winning although with different linear/nonlinear trends. In some of the plots the utility function is decreased with the number of iterations. In the context of the simulation, the decreasing utility means that the players' fatigue or lack of synergy has started to affect their performance negatively. In a real game, this could translate to players becoming tired or losing coordination with their teammates, resulting in a decline in their performance. It could also mean that the opposing team has started to play more effectively or that the strategies employed by the team are not working as intended. Ultimately, the decreasing utility indicates that the team needs to make adjustments or find ways to improve their performance if they want to continue to be competitive in the game.

To have a better sense of the effect of different parameters including synergy, fatigue and learning rate on the outcome of the game, the heat maps of  $c, k$ , learning rate and utility are plotted in Figure [3.](#page-7-0) The three heat maps provide valuable insights into the optimal parameter values for maximizing the utility of each team in the simulation.



<span id="page-5-0"></span>Figure 2: Changes in the utility function with the number of iterations for different values of  $c, k$ , and learning rate.

The first heat map of Fig [3](#page-7-0) (left top/bottom) shows that when the learning rate between the players is fixed, the highest utility is achieved when the value of fatigue is small and the synergy is large. This suggests that players who are well-rested and work well together are more likely to achieve the highest utility in the game. In the context of a real game, this may mean that teams should prioritize rest and effective communication between players in order to maximize their chances of success.

The second heat map (middle top/bottom) shows that when the synergy rate between the players is fixed, the highest utility is achieved when the value of fatigue is small and the learning rate is large. This implies that players who are both wellrested and able to learn quickly are more likely to achieve the highest utility in the game. In a real game, this may mean that teams should focus on both physical and mental preparation to optimize their performance.

In the third heat map (right top/bottm), the highest utility is achieved when the fatigue rate is fixed, the learning rate is small, and the synergy is also small. One possible interpretation of this result is that when players are fatigued, their individual abilities may be limited, and thus effective communication and teamwork become more important. In this case, a smaller learning rate may suggest that players should focus more on executing known strategies and relying on their teammates' inputs, rather than trying to learn and adapt on the fly. Similarly, a smaller synergy may suggest that players should avoid making assumptions about their teammates' actions and focus more on communicating and coordinating with each other. Therefore, in situations where fatigue is a factor, prioritizing effective communication and teamwork over individual learning ability and assumptions about teammate actions may lead to higher utility in the game.

Overall, the heat maps provide valuable insights into the complex interplay between fatigue, synergy, learning rate and utility in team-based simulations. By understanding the optimal parameter values for maximizing utility in different scenarios, teams can make more informed decisions about how to prepare for and approach real games. We believe that our results can have practical implications for designing effective multiagent systems in real-world scenarios.



<span id="page-7-0"></span>Figure 3: Heat maps of the effect of synergy, fatigue and learning rate on the utility function (team1: top, team2: bottom).

#### **4 Conclusion**

In this study, we have analyzed the impact of physical factors on the outcome of tug of war contests. We identified nine key factors that can influence the winning chances of a team. Our model assumed that teams with heavy weighted individuals, strong hands, equal heights, physical strengths and similar techniques would have an equal chance of winning, resulting in a perfectly symmetrical game.

However, by incorporating the concept of synergy and optimization theory, we were able to uncover another crucial factor that can determine the outcome of the game. Our simulations showed that the team with proportionate force distribution among its members had a higher chance of winning, even if the opposing team had equal physical attributes. Moreover, our optimization-based approach provided insights on the optimal values of fatigue, synergy and learning rate, which can be used to enhance team performance.

Our findings have practical implications for teams, coaches and players, as they can help in developing effective strategies that prioritize proportionate force distribution and reduce power wastage. Additionally, the identified optimal values can be used as guidelines to fine-tune team performance in a dynamic game setting. Overall, our study emphasises the significance of considering both physical and strategic factors in the outcome of tug of war competitions, and highlights the potential of optimizationbased approaches in modelling and analysing complex team-based games.

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# **Conflict of Interest**

The authors have no conflicts of interest to disclose.

# **Appendix: Influence of skeletal muscle characteristics on athlete's performance**

Skeletal muscle is generally classified into two types: type 1 and type 2 muscle fibres. These fibres have different characteristics and functions and are also designated by various other names. Table [1](#page-8-0) summarizes some of the key characteristics of both types of skeletal muscle [\[8,](#page-11-7) [16\]](#page-11-8).



<span id="page-8-0"></span>

The two types of skeletal muscle are referred to as type 1 and type 2 muscle fibres. Type 1 muscle fibres are also known as "slow twitch" or "oxidative muscle", while type 2 muscle fibres are known as "white", "fast twitch", or "glycolytic muscle". These different designations are based on the functional characteristics of each muscle type. Type 1 muscle fibres are typically used for activities lasting several minutes or more, while type 2 muscle fibres are used for activities lasting a minute or less [\[8,](#page-11-7) [14\]](#page-11-9).

Because of high content of mitochondria, slow twitch muscles are able to be active for a long time (high endurance). This energy production pathway is called oxidative or aerobic pathway that takes place inside mitochondria (and produces to the tune of 32 ATP (adenosine 5-triphosphate) moles per mole of glucose consumed). These muscles possess high capillary density and myoglobin content (Table [1\)](#page-8-0). The myoglobin

has  $Fe^{2+}$  ions, which explains the red coloration of type 1 muscles, and functions to trap oxygen in muscle fibres (essentially providing an oxygen reservoir). In contrast, fast twitch muscles contain high concentration of glycolytic enzymes, and so, are able to produce ATP (only a net 2 moles per mole of glucose) but quickly anaerobically, that is, without the consumption of oxygen. (Clearly the price of speed in producing ATP in anaerobic pathway is the low yield of ATP compared with the oxidative pathway which is much slower) (Table [2\)](#page-9-0).

The fast and strong contraction in type 2 muscles is also mediated through its extensive sarcoplasmic reticulum that release  $Ca^{2+}$  in milliseconds. Because of high content of mitochondria, slow twitch muscles are able to be active for a long time (high endurance). This energy production pathway is called oxidative or aerobic pathway that takes place inside mitochondria (and produces to the tune of 32 ATP (adenosine 5 triphosphate) moles per mole of glucose consumed). These muscles possess high cap-illary density and myoglobin content (Table [1\)](#page-8-0). The myoglobin has  $Fe<sup>2+</sup>$  ions, which explains the red coloration of type 1 muscles, and functions to trap oxygen in muscle fibres (essentially providing an oxygen reservoir).

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<b>Source of ATP</b>	Moles of ATP/min $\vert$ Time	
Phosphocreatine system		$8-10s$
Glycolytic system	2.5	$1.3 - 1.6$ min
Oxidative system		Unlimited time

<span id="page-9-0"></span>Table 2: Characteristics of ATP production system for muscle contraction.

Hydrolysis of ATP to ADP (adenosine diphosphate) provides the energy for muscle contraction. It is often practical to measure energy consumption in muscle tissues not in kilojoules (or kilocalories) but rather in terms of moles of ATP consumed in the process. The hydrolysis of one mole of ATP, under physiological conditions, releases approximately 50-60 kJ [\[1\]](#page-10-3). Metabolic power is then measured in terms of energy consumed or produced (in moles of ATP) per unit of time which can be readily be converted to the usual thermodynamic units of power, that is, kcal or kJ per unit of time.

During active contraction, ATP is consumed for two main processes: 1) ATPase activity in myofibrils ( $\sim$ 70%) and 2) ionic (especially Ca<sup>2+</sup>) transport ( $\sim$  30%). Under physiological conditions (35  $\degree$ C), by micro-chemical method, analysis on single human muscle fibres have shown that the rate of ATP consumption is 1.2 mM/s in slow and 2.4 mM/s in fast muscle fibres during 32 s of electrically induced intermittent contractions [\[7,](#page-11-11) [20\]](#page-12-7).

As it is explained above, the existing ATP in muscle is sufficient only for maximum a few second. Therefore, to supply more ATP for muscle activity, other sources of ATP production are needed. There are three mechanisms that provide more ATP for muscle contraction. The first source for resynthesize of ATP is the phosphocreatine (PCr) that possesses high-energy phosphate compounds. As soon as contraction starts, and to avoid the accumulation of ADP and to continue the contraction, the stored energy in phosphate bonds converts ADP to ATP (ADP + PCr  $\leftrightarrow$  ATP + Cr) [\[25\]](#page-12-8).

In glycolysis (the second source), glucose is catabolized to pyruvate releasing two ATP per glucose (Table [2\)](#page-9-0). The glycolytic pathway is anaerobic and does not require to oxygen. The resultant pyruvate then enters into mitochondria for third (and major) source of ATP production through the Krebs Cycle and the following oxidative phosphorylation. This aerobic pathway produces an average of ca. 32 ATP per glucose. The oxidative pathway is not a fast way to produce ATP but it has very high thermodynamic efficiency and yield of ATP (compared to glycolysis).

Slow muscles generate their ATP supply by oxidative phosphorylation in mitochondria and they have the ability to maintain contractile activity for a long time without showing fatigue (Table [2\)](#page-9-0). In contrast, fast muscles utilize glycolytic processes to generate ATP rapidly but this has a limit to the duration of contractile activity [\[14\]](#page-11-9).

From [\[3,](#page-10-4) Figure 9], one can surmise that during a 10-second of maximal intensity exercise, 53% of energy (in kcal/kg/min) is provided by phosphocreatine system,  $44\%$ by the glycolytic system and 3% by the aerobic system (or mitochondrial respiration) whereas, during a 30-seconds of maximal exercise, 23% of the energy comes from phosphocreatine, 49% from glycolytic and 28% from aerobic systems [\[3\]](#page-10-4). Assuming the duration of a normal tug of war contest is 60 second, the contribution of energy for phosphocreatine, glycolytic and mitochondrial respiration is 5%, 42% and 53%, respectively.

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