PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by 29 February 1972 will be published in the next issue, Vol 8 No 1, 1972. However the names of authors of successful solutions received between 1 March 1972 and the actual date of publication of Vol 8 No 1 will be published as late solutions in Vol 8 No 2. Send all solutions to the Editor (address on inside front cover).

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J161 Find a whole number, N, satisfying the following conditions:

(a) N is the product of exactly four distinct prime numbers.
(b) The decimal notation for its square is abc, def, abc where $a \neq 0$ and where the three digit number def is exactly twice the three digit number abc.

J162 Prove that for every positive whole number n,

$$(n+1)(n+2)(n+3) \ldots (2n-1)(2n) = 2^n.1.3.5. \ldots .(2n-1).$$

J163 Given the equations $x/y = x-z$, $x/z = x-y$ prove that $y = z$ and that x is not between 0 and 4.

OPEN

O164 A triangle is divided into two figures by a straight line through the centroid. Show that the ratio of the area of the smaller piece to that of the larger may take any value between a minimum of 4/5 and a maximum of 1.

O165 There are two prime numbers p such that $(2^{p-1}-1)/p$ is a perfect square. Find them, and prove they are the only ones.

O166 C_1 and C_2 are any two non-intersecting circles. P is any point on C_1, Q any point on C_2. Prove that the distance PQ is least when both P and Q lie on the line of centres. You should consider the two cases: C_1 internal to C_2, C_1 external to C_2. The shortest and most elegant proof is sought.

<u>0167</u> Show that for the angles A, B and C of a triangle,

$$\tan^2\tfrac{1}{2}A + \tan^2\tfrac{1}{2}B + \tan^2\tfrac{1}{2}C \geq 1.$$

<u>0168</u> The following observations were made at a meeting of delegates of clubs at a school:

1. For every pair of clubs represented there was exactly one person present who was a member of both.

2. Each person present was a member of at least two different clubs.

3. The chess club sent exactly three delegates.

How many people were present? How many clubs were represented?

<u>0169</u> A rectangular sheet of paper is ruled off into m horizontal rows and n vertical columns. On each of the mn squares so formed is marked either a nought or a cross in such a manner that each row and column contains both noughts and crosses. Prove that there are four squares, a, b, c, d, such that a, b lie in one column and c, d in another while a, c lie in one row and b, d in another row (i.e. they are the corners of a possibly smaller rectangle) and, either a and d are crosses and b and c are noughts, or a and d are noughts and b and c are crosses.

<u>0170</u> This puzzle in pure logic is due to Raymond Smullyan of Princeton University. What a nasty mind he must have! We shall ask another of his brainteasers in the next issue. On Christmas Day 1970, three married couples celebrated by dining together. Each husband was the brother of one of the wives; that is, there were three brother-sister pairs in the group. Helen was exactly 26 weeks older than her husband who was born in August. Mr White's sister was married to Helen's brother's brother-in-law. She (Mr White's sister) married him on her birthday which was in January. Marguerite White was not as tall as William Black. Arthur's sister was prettier than Beatrice. John was fifty years old.

What was Mrs Brown's first name?

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<u>Try this on your friends - or enemies</u>

Bertrand Russell, while at Cambridge, had a card printed on both sides with the sentence: "The statement on the back of this card is false".

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