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MATHEMATICAL GAMES

This is a new section in Parabola in which we intend to introduce you to some games (both old and new) which have a mathematical flavour. Please do not try them out in lesson-time!

Grundy's Game

This game is played with a heap of matches (any number will do) and is played by two persons. The first player divides the heap into two unequal heaps (of any number provided they are unequal) and then the second player divides either of these heaps into two unequal heaps. The game continues in this way, each player dividing any heap into two unequal heaps, until there are only heaps of 1 or 2 matches remaining (and so no-one can go). When this happens, the last person to have moved wins the game.

To see how this works, let us imagine two people called A and B playing with a heap of 9 matches. Then A divides this heap into two heaps with 7 and 2 matches, B divides the 7 giving heaps of 6, 2, 1. The game continues:

A moves to 4, 2, 2, 1;
B moves to 3, 2, 2, 1, 1;
A moves to 2, 2, 2, 1, 1, 1

and so A wins.

In the above game, A was actually able to force a win. See if you can work out why. When you have done this, see if you can work out who would win if they are started with a heap of 10 matches, or a heap of 11 matches, etc. P.M. Grundy has been able to give a set of numbers which not only tell you who wins a given game but even how to win it.

Kenyon's Game

Mr J. Kenyon of Canada has investigated a similar game to Grundy's game. In his game, each player must divide a heap into two heaps whose size differs by at least two. (In Grundy's game, they differ by at least one.) Notice that this means that a game finishes when there are only heaps of 1, 2 or 3 matches remaining. Just as Grundy did, Mr Kenyon has found a set of numbers which tell you who wins the game and the best strategy for winning it. However, in the case of Kenyon's game, there is a simple rule for remembering these numbers.

Next time, we will give you the method Grundy and Kenyon used to get their numbers. Meanwhile see what you can do yourself!

R. James.

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<u>Solutions to Follow up Problems: Pythagoras and all that, Vol 7 No 3</u>

1. All odd squares look like this: $(2k+1)^2 = 4k^2 + 4k + 1 = 4m + 1$ so $4n + 3$ cannot be a square.

2. $(a^2+b^2)(d^2+e^2) = (a^2d^2 + b^2e^2) + (a^2e^2 + b^2d^2)$

$$= (a^2d^2 + 2abde + b^2e^2) + (a^2e^2 - 2abde + b^2d^2)$$

$$= (ad + be)^2 + (ae - bd)^2.$$

3. Basically a repetition of the type of trick used in (2) only there are a lot more cross products to vanish. If you're interested, write in and ask for solutions, or else try to work it out yourself!

4. No number n of form $8h + 7$ can be sum of two squares (see (1) above.). If it was a sum of 3 squares they would all have to be odd to give $n \equiv 3 \pmod 4$. But $4k^2 + 4k + 1 = 4.k(k+1)$ and one of k and $(k+1)$ must be even. That is, m is even in (1) so all odd squares look like $8r+1$ and the sum of three will be of form $8h+3$ <u>not</u> $8h+7$.

5. If $c = a^2 + b^2$, $2c = (a+b)^2 + (a-b)^2$. If $2c = d^2 + e^2$ both d and e are odd or both are even. In either case $(d+e)$ and $(d-e)$ are each even. Then $c = \frac{1}{2}(d^2+e^2) = \frac{1}{4}(d+c)^2 + \frac{1}{4}(d-c)^2 = f^2+g^2$, f, g integers.

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<u>Do it in your head!</u>

Two trains are travelling between Sydney and Albury (which are 640 km apart). Train A leaves Sydney at 9.00 am, and travels at 80 km per hour towards Albury and train B leaves Albury at the same time and travels at 100 km per hour. Calculate in your head how far paart the trains will be one minute before they pass one another.

(Answer on page 40)

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