RESEARCH CORNER

The "Four 4's" question last year provoked so much interest that we have decided to dig up a few more "research problems", i.e. questions which may have no final answer but where you can try to get as far as possible. We hope to have one research problem per year and print a list of those who have gone furthest with the problem. Let us know what you think of this and if you have any suggestions.

This year's problem is closely related to the article "Pythagoras and all that" in Vol 7 No 3. In that article, you were told that every number can be written as the sum of four or fewer perfect squares and our problem is: Which numbers can be written as the sum of 2 squares, which require 3 squares and which require 4 squares? For example, $7 = 4 + 1 + 1 + 1$ requires 4 squares, but $8 = 4 + 4$ only requires 2, and $11 = 9 + 1 + 1$ requires 3.

You might also like to find all the different ways a number can be written with as few as possible perfect squares. For example, all the numbers up to 26 can be written in only one way ($5^2 + 1$ and $1 + 5^2$ are, of course, the same) but $27 = 5^2 + 1 + 1 = 3^2 + 3^2 + 3^2$ and 28 can be written as the sum of 4 squares in 3 different ways. Let us know how far you get - some people might like to try to find some general rules!

Incidentally, the best attempt at the four 4's was by David Adams of Blayney High who has managed to reach 200 with gaps at 157, 167, 173 and 187. Those interested might like to write to us and we will see if we can get his complete list for you.

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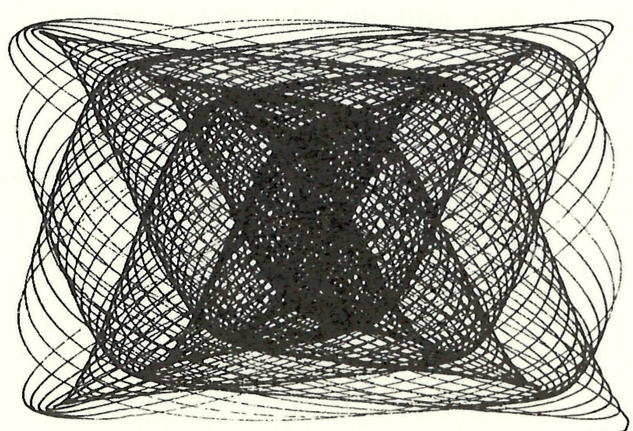

A Table with corridors and squares

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Here is a perfectly ordinary set of multiplication tables up to 10×10, but with walls built in enclosing corridors of numbers. Thus one corridor contains 2, 4 and 2 and so on. Try adding up the numbers in the corridors and see what you get. Why?

Now take any wall and add up all the numbers it encloses to its left and above it. Thus one wall encloses 1, 2, 2, 4. Again there's a pattern to your answers - why?

Solutions next issue but try to beat us to it by sending in your own explanations to the Editor as soon as possible. We will print them if we can. (Solutions from 1st and 2nd form students will get preference - Ed.)