PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by 23 June 1972 will be published in the next issue Vol 8 No 2, 1972. However the names of authors of successful solutions received between 24 June 1972 and the actual date of publication of Vol 8 No 2 will be published as late solutions in Vol 8 No 3. Send all solutions to the Editor (address on inside front cover).

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

JUNIOR

J171 Prove that for $n > 2$,

$$(n!)^2 > n^n.$$

J172 There are certain pairs of integers x, y for which $2x + 3y$ is divisible by 11, e.g. $x = 1$, $y = 3$. Prove that for exactly the same pairs of integers also $7x + 5y$ is divisible by 11.

J173 The square of a number X has the digit 7 at the tens. What is the last digit of X^2?

OPEN

O174 How many n-digit numbers can one form from the digits 1, 2, 3 alone, so that each of the three digits should appear at least once in the number?

O175 Let $a^2 + b^2 + c^2 = 1$. Prove that

$$-\tfrac{1}{2} \le ab + bc + ca \le 1.$$

0176 Prove that if we are given any n positive integers $a_1, a_2, \ldots, a_n$ there are always integers k, m, $1 \leq k \leq m \leq n$ so

$$a_k + a_{k+1} + \ldots + a_{m-1} + a_m \text{ is divisible by } n.$$

e.g. given 13, 5, 18, we have 13 + 5 (= $a_1 + a_2$) divisible by 3; we also have 18 = a_3 divisible by 3, so that in the last example k = 3 = m.

0177 (i) D, E, F are points on the line segments BC, CA, AB such that the circles through CDE, BDF intersect again at the point P in the interior of the triangle ABC. Prove that the circle through A, E, F also passes through P.

(ii) If D lies on BC produced so that DEF is a straight line prove that P also lies on the circumcircle of the triangle ABC.

0178 Prove the correctness of the construction given for obtaining $\sqrt[3]{2}$ on page 10 of this issue.

0179 This is the promised puzzle due to Raymond Smullyan. Three men A, B and C are all known to be perfect logicians; that is, when presented with any body of premises all the valid consequences are instantaneously clear to them.

X They are shown a set of 8 markers, 4 black and 4 white, and then blindofolded. Two markers are attached to the forehead of each man and the remaining two are concealed before their blindfolds are removed.

A is asked if he knows the colours of the markers he bears. "No", he says. The same answer is elicited in turn from B, from C and then a second time from A. Now B announces that he knows the colours of his markers. What are they? Explain.

0180 Joe brought $2500 with him to a two-up club and employed the following system of betting. He started with one dollar and trebled the bet after each loss and started again with $1 after a win. All the money he won plus the capital in that win, he put into a bag. After having won his last bet and gone through the $2500 exactly, he went to celebrate and mislaid the bag which was found by an honest club manager. The manager then announced that he would give the bag to the person who knew the exact amount in the bag because there were several claimants. All Joe could remember was that he had won exactly 100 bets but after consulting a mathematician-friend, the amount was identified and the bag returned to Joe. How much was there in the bag? (This problem was sent in by Mr J. Tong, an ex school Parabola reader - C.D. Cox.)