FROM OUR READERS

Dear Sir,

The sum of the numbers in a corridor (A Table of Corridors and Squares, Vol 8 No 1) can be represented as follows (n is the number at the top of the corridor):

$$2[n + 2n + 3n + \ldots + \{(n-1)n\}] + n^2$$
$$= 2n[1 + 2 + 3 + \ldots + (n-1)] + n^2$$
$$= 2n\left\{\frac{(n-1)n}{2}\right\} + n^2$$
$$= n^2(n-1) + n^2$$
$$= n^3 - n^2 + n^2$$
$$= n^3.$$

As n is a whole number, the sum of the numbers is a perfect cube.

The sum of the numbers in a square can be represented by (n is the number at the head of the outer corridor):

$$(1 + 2 + 3 + \ldots + n) + (2 + 4 + 6 + \ldots + 2n) + (3 + 6 + 9 + \ldots + 3n) + \ldots + (n + 2n + 3n + \ldots + n^2)$$
$$= 1(1 + 2 + 3 + \ldots + n) + 2(1 + 2 + 3 + \ldots + n) + 3(1 + 2 + 3 + \ldots + n) + \ldots + n(1 + 2 + 3 + \ldots + n)$$
$$= (1 + 2 + 3 + \ldots + n)(1 + 2 + 3 + \ldots + n)$$
$$= (1 + 2 + 3 + \ldots + n)^2$$

which is a perfect square.

Therefore

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2.$$

Alan Fekete,
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Consecutive Digits

The number 12784 has the remarkable property that the number formed by the first two digits, when multiplied by the third digit, yields the number formed by the last two digits (12 x 7 = 84). However, 1,2,4,7,8 are not consecutive digits. Can you find a similar number using consecutive digits? (not necessarily in the right order, of course).

(Answer on page 36)