FROM OUR READERS

Dear Editor,

In issue 3 of 1971 you gave a short note about a card with "The statement on the back of this card is false," printed on either side.

If you pick one side and assume that statement to be true, that makes the other statement false in your eyes. As the other statement says the first statement is false, the other statement is indeed false from your viewpoint.

Therefore if you pick one side the statements are consistent and pose no problem.

However if a card is provided with "The statement on the back of this side is false" on one side and "The statement on the back of this side is true" on the other, you get a situation in which the statements are indeed inconsistent.

Doug McLeod,
Beacon Hill High School

Criticism to: "There are no uninteresting numbers" (Vol 8 No 1)

Dear Editor,

The basic fault in reasoning is that $\overline{S}$ (the set of interesting numbers) is NOT well ordered, and therefore has no lowest element, and in the same way no largest element. Consequently it is impossible to determine the smallest and largest element in S.

Jerry Schwartz,
(Cranbrook)

Dear Sir,

A solution to the trisection on page 14, Vol 8 No 2 is:

$$\theta + \angle AOC = 180^\circ$$
$$\phi + \angle AOC + \angle CAO = 180^\circ$$
$$\therefore \theta = \phi + \angle CAO.$$

By joining B to O one forms two isosceles triangles, ΔCBO and ΔABO. Thus $\angle BAO = \angle ABO$ and $\angle ACO = \angle BOC$. As exterior angles = the sum of the two opposite interior angles $\angle BAO = \angle ABO = \angle BCO + \angle COB$ or $\angle BAO = 2\phi$.
$\therefore \angle BAO + \phi = 3\phi = \theta$ is proved. (See Diagram 2 on page 14 of Vol 8 No 2 – Ed.)

Yours,

Richard Young,
1st Form, Caringbah High

Dear Sir,

In the first short problem on page 24 of Vol 8 No 2 we are asked to rotate a 4 row equilateral triangle by moving 3 dots. If we try to find the minimum number of dots to be moved to *invert* an n-row triangle, and use the notation (p,q) to represent the p'th dot, in the q'th row from the left, (similar to K. Burns' notation in "Triangle Similar to Pascal Triangle") and regard the apex as (1,1) and consider where n is odd, interesting results are achieved.

If the apex of the new triangle is placed in the middle of the base of the old (at $(\frac{n+1}{2},n)$), $\frac{n-3}{2}$ more dots are needed than if the new apex is placed at $(\frac{n+1}{2},n+1)$.

Alan Fekete,
Form 1, Sydney Grammar School

The Graph of $y = (-1)^x$

The most obvious thing about this graph is that it does not have meaning in the real number system for some values of x i.e. $(-1)^{1/2} = \sqrt{-1}$. If x is written as $\frac{m}{n}$ where m,n are integral, $n \neq 0$, then it can be immediately seen that if n is even the function may be written $(-1)^{m/2p}$ where p is integral.

$$\begin{aligned}(-1)^{m/2p} &= ((-1)^{1/2p})^m \\ &= (((-1)^{1/2})^{1/p})^m \\ &= (\sqrt{-1})^{m/p}\end{aligned}$$

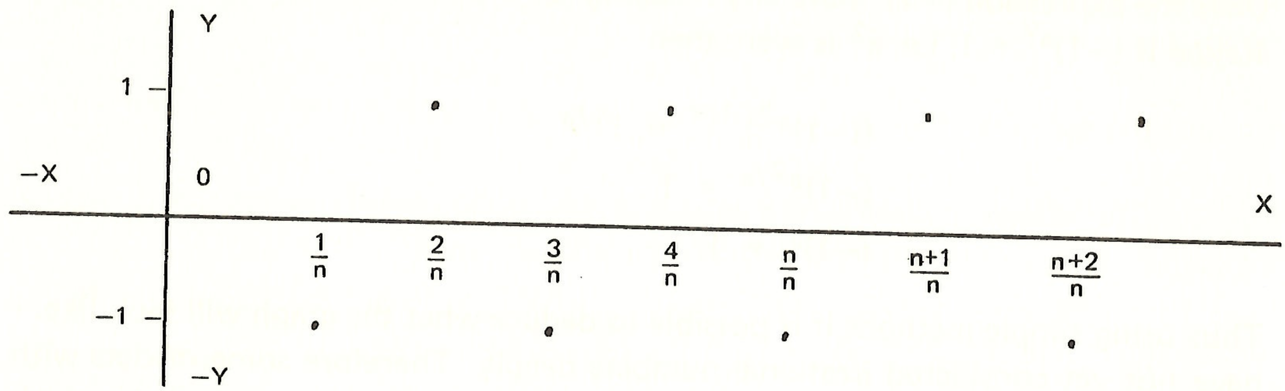
which has no value in the real number system. However if n is odd the graph is $y = ((-1)^{1/n})^m$.

Now $(-1)^{1/n}$ must be equal to -1 so

$$y = (-1)^m.$$

If m is even, $(-1)^m$ must equal 1. (If $m = 2p$, $(-1)^m = (-1)^{2p} = ((-1)^2)^p = 1^p = 1$) and if m is odd, $(-1)^m = -1$. (If $m = 2p + 1$, $(-1)^m = (-1)^{2p+1} = ((-1)^2)^p \times (-1)^1 = 1^p \times -1 = -1$).

Therefore for any odd denominator n, an infinite number of values $(-1)^{m/n}$ may be obtained by substituting different values of m. Each consecutive value will be on the opposite side of the X axis to the last one. i.e.



By picking enough odd n the points along $y = \pm 1$ will become dense — but of course not full.

The meaning of $(-1)^{-m/n}$

$(-1)^{-m/n} = \frac{1}{(-1)^{m/n}}$. Now if $(-1)^{m/n} = -1$, then

$$\begin{aligned}\frac{1}{(-1)^{m/n}} &= \frac{1}{-1}\\ &= -1\\ &= (-1)^{m/n}.\end{aligned}$$

Also if $(-1)^{m/n} = 1$, then $\frac{1}{(-1)^{m/n}} = \frac{1}{1} = 1 = (-1)^{m/n}$.

Thus $(-1)^{-m/n} = (-1)^{m/n}$.

This is what would be expected from the graph to maintain the zigzag pattern; note $(-1)^0 = 1$.

Proof That $(-1)^{m/n}$, where m,n are integral, $(-1)^{m/n}$ must equal either −1 or 1. (n is odd.)

Let $(-1)^{m/n} = a$.

$(-1)^{m/n} = a$

$(-1)^{(m-2)/n} \times (-1)^{2/n} = a.$

$(-1)^{(m-2)/n} \times 1 = a.$

$(-1)^{(m-4)/n} \times (-1)^{2/n} = a.$

$(-1)^{(m-4)n} = a.$

This process is continued, until we get $(-1)^{1/n} = a$, i.e. $a = 1$ or -1, or $(-1)^0 = a$.

Does the expression $(-1)^a$ have any meaning for irrational numbers a?
Maybe if $(-1)^{a^2} = 1$, i.e. a^2 is even, then

$$((-1)^{a^2})^{1/a} = 1^{1/a}$$
$$(-1)^{a^2/a} = 1$$
$$(-1)^a = 1.$$

Thus using simple methods it is possible to deduce what the graph will look like. I have not yet considered irrational numbers deeply. Therefore some readers with nothing to do one Friday night might try to see if they can extend the graph further in this direction.

Doug McLeod,
Beacon Hill High School

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Archimedes discovers his law