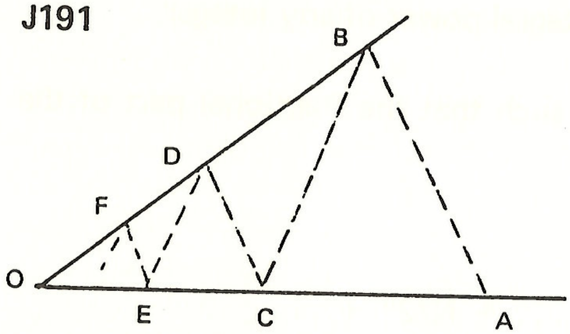
PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by January 31, 1972, will be published in the next issue Vol 9 No 1, 1973. Send all solutions to the Editor (address on inside front cover).

Only those students who have not yet commenced their fourth year of secondary education are eligible to submit solutions of problems in the Junior Section. All students may submit solutions of problems in the Open Section.

Junior

J191



A yacht starts from a point A, one mile due East of a buoy O, and tacks up to the buoy as indicated in the diagram. First, it sails on the course AB, 30° West of North, until the buoy bears exactly South-West; then it tacks on the course BC, 30° West of South until the buoy's bearing is again due West. This process is repeated indefinitely as indicated by the broken line ABCDEFG. . . . After reaching the buoy, one of the deck hands (who wishes to take his mind off the incredible exertions of the last few feet of the trip) calculates how far they have actually sailed. Can you?

J192 Replace each E by an even digit (0, 2, 4, 6 or 8) and each O by an odd digit (1, 3, 5, 7 or 9) so that the following multiplication is correct.

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   E E O
     O O
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 E O E O
  E O O
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O O O O O
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J193 If $1 \leqslant a \leqslant 2$, simplify $\sqrt{[a+2\sqrt{(a-1)}]} + \sqrt{[a-2\sqrt{(a-1)}]}$.

O194 Twenty four coins are of the same size and external appearance; some are pure gold, and the rest are of a lighter alloy. Using at most thirteen weighings on a pan balance, how can we determine how many coins are gold?

O195 Four different buttons are placed on the numbers 1, 2, 3 and 4 of a horizontal clock face. A button may be moved in either a clockwise or an anticlockwise direction over four other numbers to a fifth number (i.e. it is moved 25 minutes one way or the other) provided its destination is not already occupied by another button. After a certain number of such moves the buttons again cover the numbers 1, 2, 3 and 4. How many different rearrangements of the four buttons are possible as a result of this process?

O196 Prove that the sum of the same even power of nine consecutive integers, the first of which exceeds 1, cannot be any integral power of any integer.

O197 Does there exist a natural number n such that the fractional part of the number $(2 + \sqrt{2})^n$ exceeds 0.999999?

O198 Prove that in the equation

$$N = N/2 + N/4 + N/8 + \ldots + N/2^n + \ldots$$

(where N is any natural number) every fraction may be replaced by the nearest whole number. (If the fraction ends in a ½, replace it, as usual, by the next whole number.)

O199 Prove that if the polynomial

$$P(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$$

assumes integral values for all integral values of x, then it is possible to express P(x) as

$$b_0 + b_1x + b_2\frac{x(x-1)}{1.2} + b_3\frac{x(x-1)(x-2)}{1.2.3} + \ldots + b_n\frac{x(x-1)(x-2)\ldots(x-n+1)}{1.2.3\ldots n}$$

where $b_0, b_1, \ldots b_n$ are integers.

O200 The hole in the head of a spanner is a regular hexagon with sides of length a. The spanner is used to tighten a square nut with sides of length b. Find conditions satisfied by a and b for this to be possible.

SOLUTIONS

Solutions to Problems 181–180 in Vol 8 No 2

The names of successful problem solvers appear after the solution to Problem 190.

Junior

J181 Show that the product of 4 consecutive integers is always one less than a perfect square.

Answer
$$\begin{aligned}(n-1)\,.\,n\,.\,(n+1)\,.\,(n+2) &= [n\,.\,(n+1)]\,.\,[(n-1)\,.\,(n+2)]\\ &= (n^2+n)\,.\,(n^2+n-2)\\ &= [(n^2+n-1)+1]\,.\,[(n^2+n-1)-1]\\ &= (n^2+n-1)^2-1\end{aligned}$$

J182 Find the integral solutions of the equation $y^3-x^3=91$.

Answer $\qquad y^3-x^3 = (y-x)(y^2+xy+x^2) = 91.$

Hence $\qquad y-x = \pm 1,\ \pm 7,\ \pm 13,$ or ± 91

and $\qquad y^2+xy+x^2 = \pm 91,\ \pm 13,\ \pm 7,$ or ± 1 respectively.

Since $(y+x)^2 = 1/3[4(y^2+xy+x^2)-(y-x)^2]$ must be positive, the values -91, -13, 7, -7, 1 and -1 of (y^2+xy+x^2) must be discarded. Corresponding to $y-x=1$, we obtain $y+x=\pm 11$, yielding solutions $(x,y) = (5,6), (-6,-5)$.

Corresponding to $y-x=7$, we obtain $y+x=\pm 1$, yielding the solutions $(x,y) = (-3,4)$ and $(-4,3)$.