THE SUPERELLIPSE AND THE SUPEREGG

If you went to Stockholm to-day, you would find in the middle of the city an "oval-shaped" pool about 200 metres long with fountains and a translucent base. Under this pool is a self-service restaurant with sunlight filtering through the pool and surrounded by shops.

But the most fascinating thing about this pool is its shape. In 1959, the Swedish town-planners were faced with the problem of what would look nicest in the rectangular area set aside for the pool. They tried an ellipse, but found that its pointed ends caused problems in the flow of traffic around the pool. They next tried a curve made up of eight circular arcs but it looked clumsy. Finally, they consulted the inventor Piet (pronounced "Pete") Hein who told them that they needed "a curve that mediates between . . . the ellipse and the rectangle." He suggested "a curve with the same equation as an ellipse but with an exponent of 2½" – which he nicknamed a *superellipse*. This curve was just right and the planners were able to use several concentric superellipses in the project, completing the work a couple of years ago.

To understand his answer, let us look at a series of graphs. We all know that if a and b are two positive numbers then the graph of $x/a + y/b = 1$ would be a straight line and so if we draw this graph and the graphs of

$$\frac{x}{a} - \frac{y}{b} = 1, \quad -\frac{x}{a} + \frac{y}{b} = 1 \quad \text{and} \quad -\frac{x}{a} - \frac{y}{b} = 1,$$

we would obtain the following parallelogram:

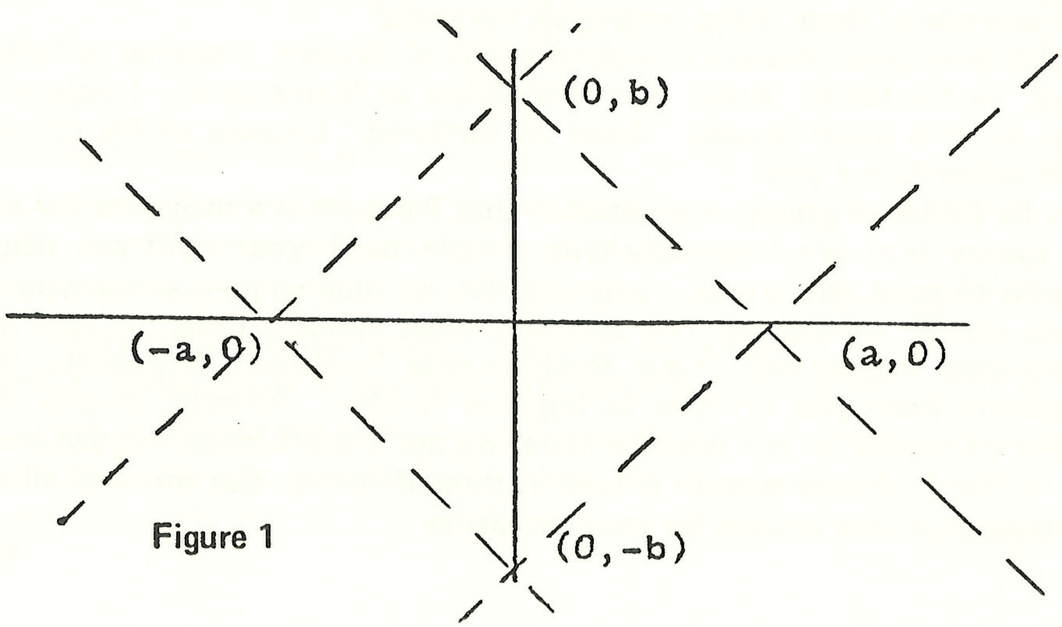

Figure 1

Because we are only interested in the parallelogram itself, we can ignore the rest of the four lines by drawing the graph of $|x/a| + |y/b| = 1$:

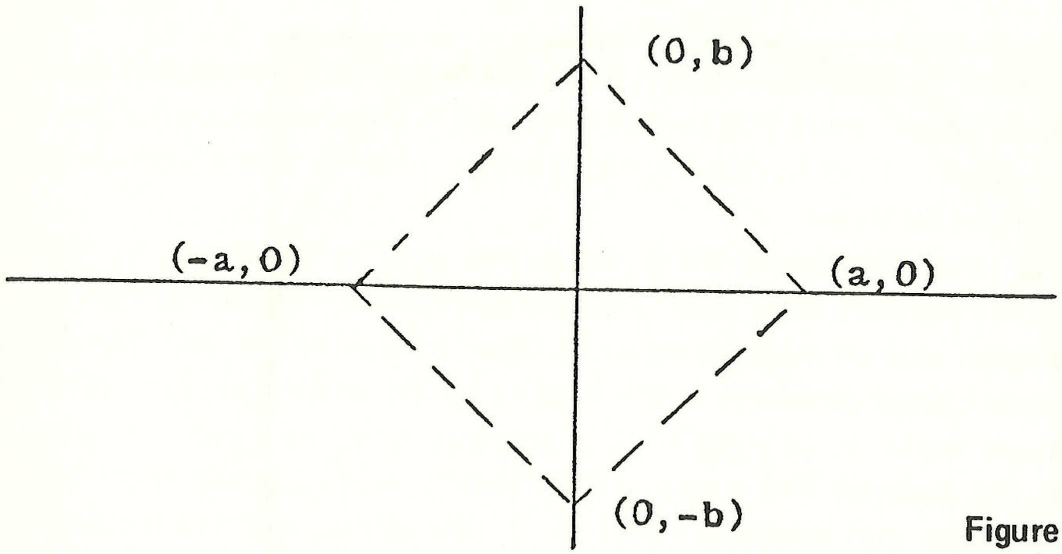

Figure 2

By plotting a lot of points, you will also be able to see that the graph of $x^2/a^2 + y^2/b^2 = 1$ is an ellipse. (This is not so strange if you know that the graph of $x^2 + y^2 = 1$ is a circle – try it!) When you have done this, you might like to draw the graphs of $|x^3/a^3| + |y^3/b^3| = 1$, $x^4/a^4 + y^4/b^4 = 1$, and so on. If you do, you will see that the graph of

$$\left|\frac{x^n}{a^n}\right| + \left|\frac{y^n}{b^n}\right| = 1$$

looks more and more like a rectangle as you use larger and larger values for n. In figure 3 some of these graphs have been drawn for the case when a = b. (In this case, the ellipse becomes a circle and the rectangle becomes a square.) So the superellipse that Piet Hein invented in the graph of

$$\left|\frac{x}{a}\right|^{2\frac{1}{2}} + \left|\frac{y}{b}\right|^{2\frac{1}{2}} = 1.$$

After its success in Stockholm, the superellipse has been used in Scandinavia as a basic shape for tables, chairs, plates, lampshades, and other household objects. Meanwhile, Piet Hein turned his attention to three dimensions. If you imagine an ellipse rotating about its longer axis, you will be able to picture the solid which mathematicians call a prolate spheroid (for example, many older atlases will tell you that the world is a prolate spheroid – or imagine an egg which is pointed at both ends). Piet Hein tried rotating his superellipse in the same way and produced what he called a *superegg* which, unlike an ordinary egg, has the remarkable property that it will balance perfectly on either end. In fact, if we made the value of n large enough in our formula, we would be able to make a "super egg" which would balance on its end and which was as narrow as we liked and as tall as we liked.

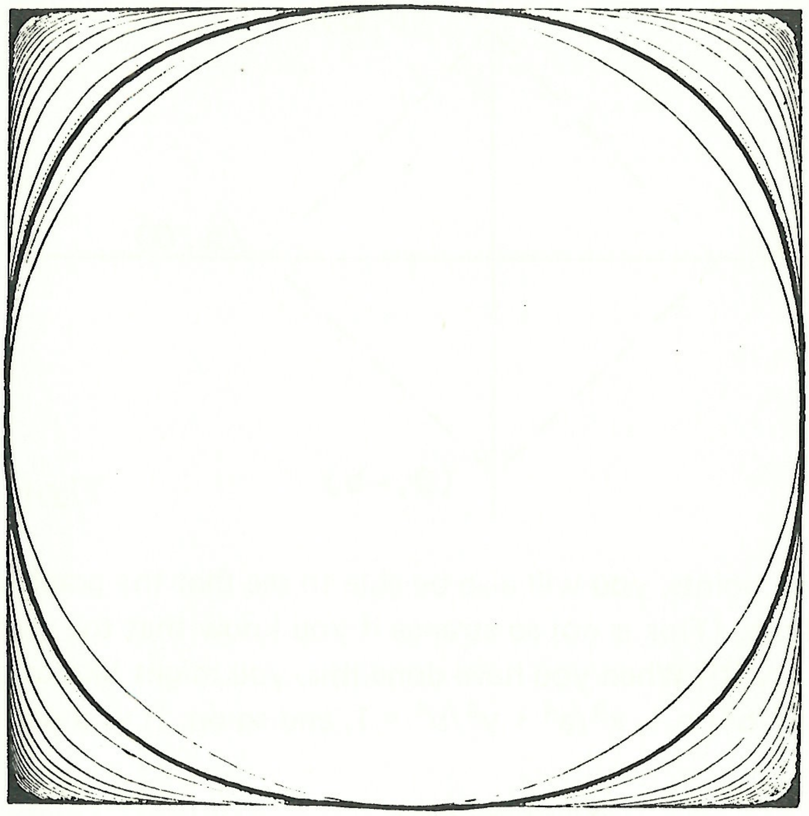

Figure 3

Problems

1. The parallelograms in Figures 1 and 2 are actually rhombuses. Why?
2. Why does the graph of Figure 2 only have segments instead of complete lines like Figure 1? (Remember that $|2| = |-2| = 2$; and in general $|x| = x$ when x is positive and $|x| = -x$ when x is negative.)
3. On a piece of graph paper, draw the graphs of

$$\frac{x^{2\frac{1}{2}}}{32} + y^{2\frac{1}{2}} = 1 \quad \text{and} \quad \frac{x^2}{16} + y^2 = 1.$$

4. By drawing the graphs of $x^n + y^n = 1$ for $n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots,$ try to guess what happens as n gets closer and closer to 0.
5. (For Seniors) What must be the ratio of the height to the width of a superegg in order for it to be stable?
6. (For geniuses) Design a better curve to solve the problems of the Stockholm planners.