PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by May 31, 1973, will be published in the next issue Vol. 9 No. 2, 1973. Send all solutions to the Editor (address on inside front cover).

Starting in this issue the problems are classified as Junior, Intermediate or Open. Only students in the first two years of secondary education are eligible to submit answers to the Junior problems, and only those in the first four years may submit answers to the Intermediate problems. Anyone may submit solutions of problems in the Open section.

Junior

J201 Find the last digit of 7^{1001}.

J202 To divide 410,256 by 4, we need only take the 4 from the beginning and put it at the end; i.e. 410,256 ÷ 4 = 102,564. Find a number starting with 8 which can be divided by 8 by taking the 8 from the beginning to the end.

Intermediate

I203 If $a > b > 0$, which of $\sqrt{a-b}$ and $\sqrt{a} - \sqrt{b}$ is the greater?

I204 (i) Prove that if it is possible to tile a rectangle with square tiles no two of which are of the same size then

(a) the smallest tile does not touch any side of the rectangle.

(b) the smallest tile touches exactly four other tiles.

(ii) Prove that it is possible to exactly fill a rectangular box with cubes no two of which are of equal size.

I205 Suppose that the number ABCDE (where the letters denote decimal digits) is divisible by 271. Prove that also BCDEA is divisible by 271. (For example: since 27371 is divisible by 271, so is 73712).

Open

O206 Prove that if n is not divisible by 3 then $n^{13}-n$ is divisible by $2^{13}-2$.

O207 ABCD is a quadrilateral such that

(i) $AC^2 + BD^2 = 2AB^2 + AD^2 + BC^2$

(ii) $AD = BC$.

Prove that it is a parallellogram.

(By courtesy of Prof. C.S. Venkataraman, Sree Kerala College, Trichur, South India.)

O208 In the following addition, replace the letters by digits so that the calculation is correct. (All occurrences of a letter must be replaced by the same digit. Different letters stand for different digits.)

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    J U N E
+   J O L L Y
+   H O L L Y
+       J O N
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  J O L Y O N
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(The possibility of this puzzle was observed by a Hungarian schoolboy, a viewer of the fine B.B.C. television series, "The Forsyte Saga.")

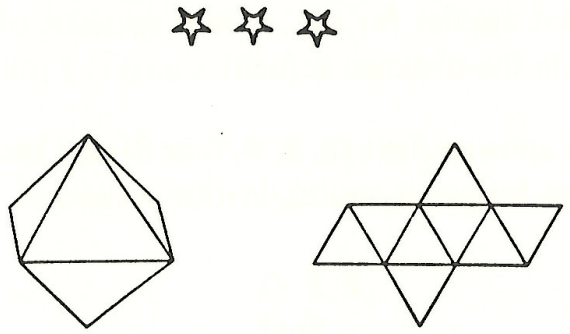
O209 An ordinary deck of cards is thoroughly shuffled and cut at random. Player A then exposes the cards from the top, one at a time, and for each card that is not an ace, B pays A 10 cents. However, as soon as an ace turns up, A pays B $1.20 and the game is finished. Is this game perfectly fair? If not, which player does it favour.

O210 *(Submitted by Malcolm Temperley, sometime Parabola solver, now at the Australian National University.)* Six houses on one side of a street are numbered

from 20 to 30, from left to right. In each lives a man with a different occupation, pet, hobby and five of them smoke a different brand of cigarette. No two men are the same age and none is over sixty.

1. Rev. Martin who does not believe in playing sport on Sundays, lives in the house next to the church.
2. The banker smokes Alpine.
3. The lawyer often complains about his neighbour's cat because of his prize pet pigeons.
4. Mr Clarke smokes Cambridge.
5. Mr Hughes is annoyed by his neighbour's loud music.
6. The 42-year-old man has a 30-year-old man to the left (to your left).
7. Dr Ferguson's neighbour is 8 years older than he.
8. Mr Sullivan trades coins *for* stamps with the man in number 28.
9. The 34-year-old man plays squash every day of the week.
10. Escort is smoked by the 53-year-old man.
11. Mr Walker lives next door to the Marlborough smoker.
12. The reporter, being the youngest, is not quite 30.
13. The executive lives in number 24.
14. Mr Hughes prefers pet goldfish to pet guinea pigs.
15. The budgies are often annoyed by the model planes next door.
16. Benson and Hedges are smoked in a house next to the house where the dog is kept.
17. The doctor lives in number 20.

Now who plays golf? Who does not smoke? And who keeps budgies? (P.S. *Medical authorities warn that smoking is a health hazard.*)



Octahedron {3, 4}