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POLYOMINOES

The term "polyomino" is used to describe a set of squares, which are simply connected together by edges. The word "domino" used in this way refers to the shape of the playing dominoes, that is two squares simply connected along an edge.

There are five different tetrominoes (4 squares). They are shown below in Figure 1.

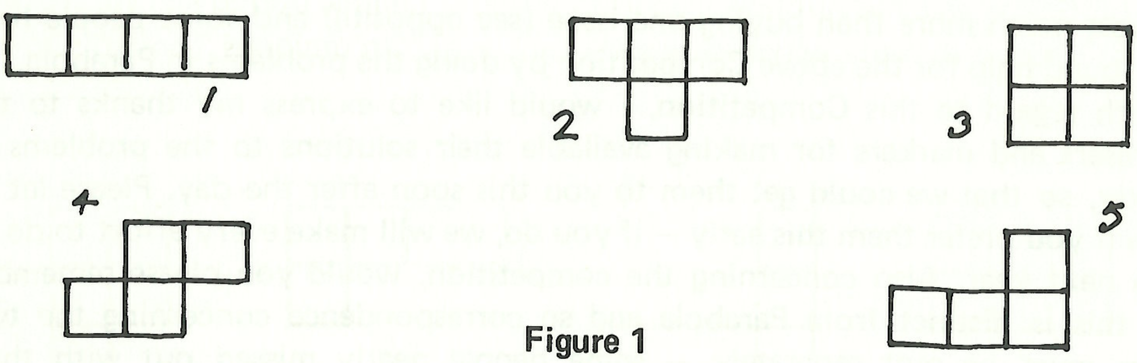

Figure 1

One can easily be excused for thinking that some tetrominoes have been omitted, for example, the two tetronominoes shown in Figure 2. But we must remember that it is only the "shape" that we are concerned with, the ways of connecting the squares together. You can see quite simply by rotating one of the tetrominoes in Figure 2 by 90°, that the two shapes are in fact the same. We say that one is a rotation of the other.

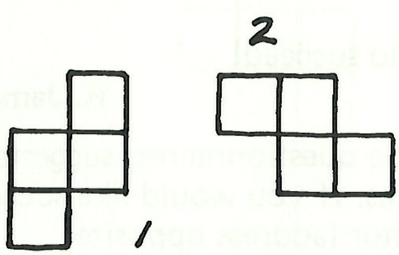

Figure 2

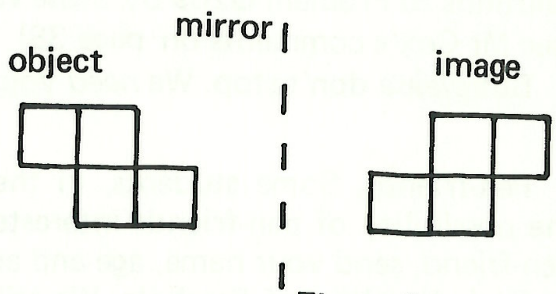

Figure 3

Similarly, a mirror placed next to the tetromino in Figure 1(4) will give the tetromino in Figure 2(2) – a similar result could be obtained by cutting out the tetromino in Figure 1(4) and then turning it over. You should also notice that no amount of rotating the tetrominoes of Figure 2 will make the tetromino in Figure 1(4) exactly.

When objects are the "same" in geometry, that is they can be fitted exactly over the top of each other they are said to be *congruent*. The operations of rotating and reflecting are called *congruence transformations*.

There are twelve distinct (non-congruent) pentominoes — they are shown in Figure 4. It is also known that there are 35 distinct hexominoes which you might like to draw yourselves, but be careful not to draw reflections or rotations of ones that you might already have drawn.

It is interesting that, as yet, the problem of how many polynominoes there are of a particular size has not been solved in general. That is, nobody as yet has been able to write a formula which gives the number of " 'n'ominoes " for all numbers n.

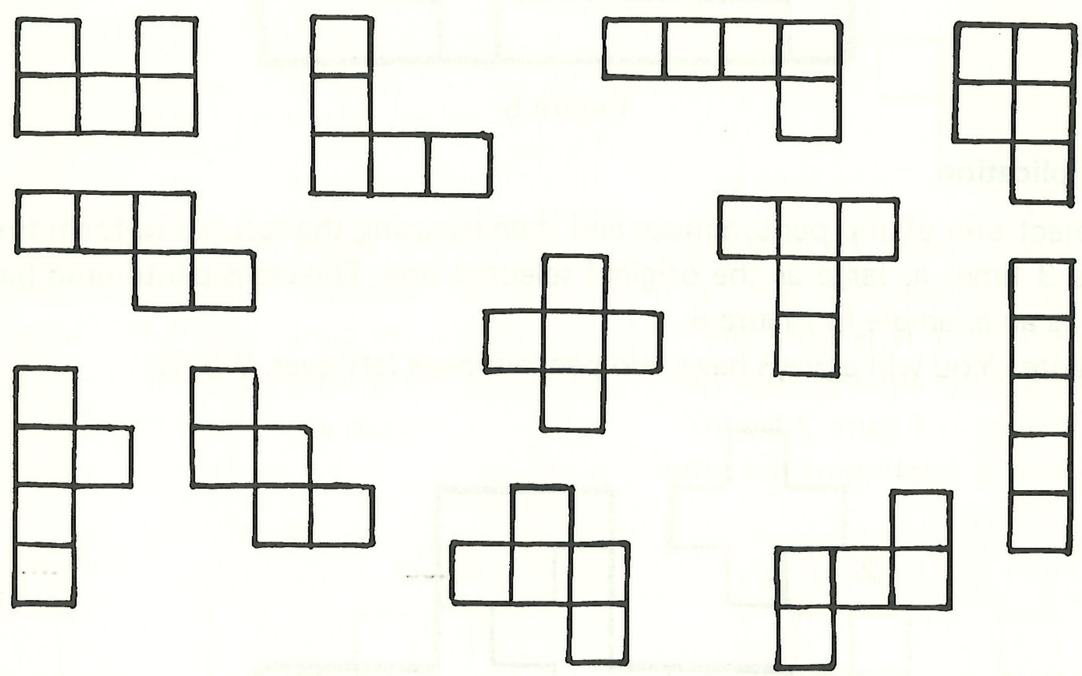

Figure 4

Problems

The way to solve these problems is to make a set of pentominoes out of cardboard.

1. Rectangles

There are twelve pentominoes, and therefore they have a combined area of 60 square units. The rectangles with 60 square units area and integral sides are 10 x 6, 12 x 5, 15 x 4, 20 x 3, 30 x 2, 60 x 1. The idea is to try and pack the pentominoes into the above rectangles as shown in Figure 5 for the 10 x 6 rectangle.

(*Note:* For the last two rectangles mentioned, it is impossible to do this. Why?)

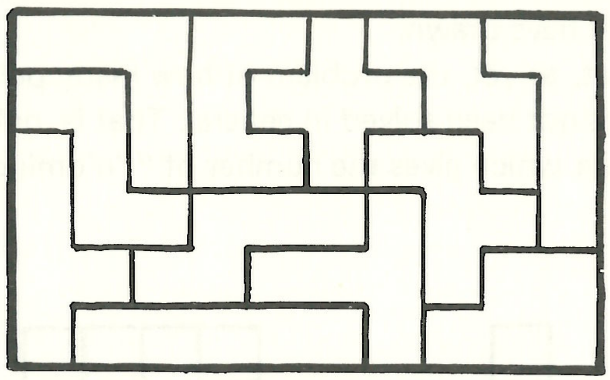

Figure 5

2. Triplication

Select one of the pentominoes and then by using the rest try to form the same shape 3 times as large as the original selected one. The cross pentomino has been used as an example in Figure 6.

(*Note:* You will always have two pentominoes left over. Why?)

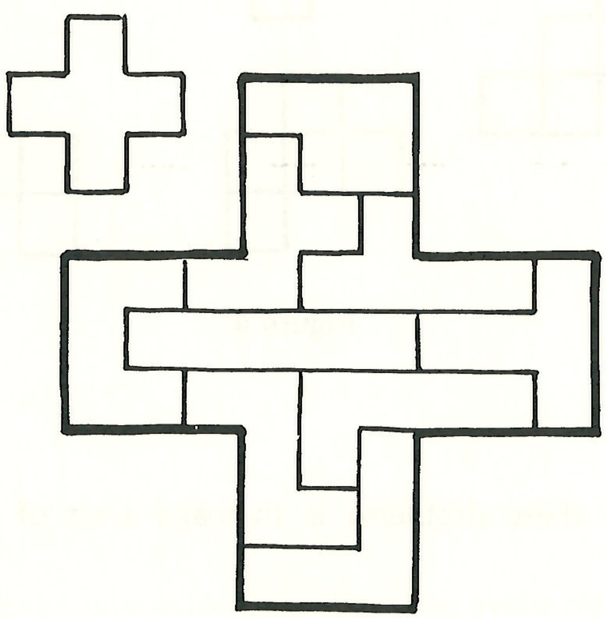

Figure 6

K. Wilkins