MATHEMATICAL GAMES

The game this time is based on the article 'Polyominoes' on page 2 of this issue.

Pentominoes

This game is played by two people on a standard chess board. You need a set of the twelve distinct pentominoes, cut out, so that the squares of the pentominoes are the same size as the squares of the chess board. Now in turn each player selects a pentomino, and places it on the chess board, aligning the squares. You may place it in any position on the board that you care to. The first player not able to place one loses.

Although the game lasts for at the most twelve goes it is very difficult to analyse because of the large number of moves possible.

Hex

Well, where are all the hexperts? Firstly as promised is a proof that the first player must be able to win. (Did you guess it would be the first player?)

1. The game of hex can never end in a draw. That is, it is impossible for all the hexagons to be taken without a winning chain being formed. To see this clearly we use the idea of separations. A winning chain will separate the two sides of the rhombus belonging to the loser. On the other hand, the loser's chains do not separate the edges of his opponent, and thus a winning chain can (and must) be formed because all hexagons are used — so we have a winner.

2. Each game has a winner, so there must be a winning strategy. The strategy could be for the first or second player. Let us assume that the winning strategy is for the second player.

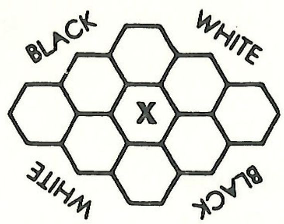
3. The first player adopts the following defence. He makes an arbitrary first move, then he pretends he is the second player, and uses the second player's winning strategy. If this strategy causes him to select the position of his first move, he makes another arbitrary move and so on.

4. The first player's extra piece in this game is always an advantage, it can never interfere with the winning strategy. The first player must therefore win. The assumption that the second player has a winning strategy is therefore false, and we are forced to conclude that the first player can always win.

This proof only shows us that there is a winning strategy for the first player; it tells us nothing about the form of the strategy.

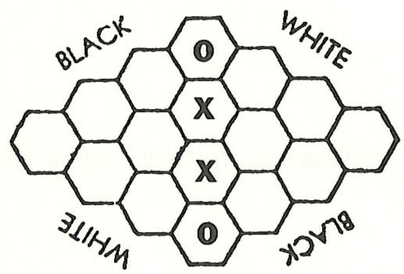
On the 3x3 board and 4x4 board winning strategies are quite easy to devise.

3x3 Board



The first player wins by taking the centre hexagon first.

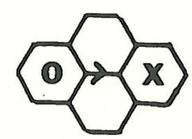
4x4 Board



The first player wins in 5 moves if he selects either of the two hexagons marked "X".

He can always be defeated if he selects any unmarked hexagon.

On the standard board the story is much different. All references we have to the game state that no winning strategy for hex is known. Barry Quinn's "system" of play, described in his letter to the Editor in this issue, is indeed quite a good system, and will defeat many players. In his system the first player immediately selects the centre hexagon. From there he endeavours to form a disconnected path to each of his sides, by the following procedure. His next hexagon is selected by moving along an edge between two vacant hexagons, as show in the diagram:



We can see that sometimes there will be as many as six different ways in which he may choose his next hexagon in this system (moving from the six vertices of a hexagon in the centre).

It would seem therefore quite difficult to prevent the first player from forming this path. When he has formed such a path he has won the game, as he has two choices to complete each section of the chain.

As we have said, this system will defeat a large number of players. However, it is not certain whether it will defeat *every* player. To convince us of this fact, Barry Quinn would have to demonstrate that his system will always work *irrespective of what the other player does*. On the other hand, to show that his system does not work, we need to find a set of moves for the second player which will counter the original system.

Can any reader decide the matter one way or the other for us?

What we are sure about however is that a winning strategy does exist for the first player.

K. Wilkins

Books which may help you

"Mathematical Puzzles and Diversions" by Martin Gardiner.
"Excursions into Mathematics" by Beck, Bleicher & Crowe.

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Crossnumber

Since the publication of Vol. 8 No. 3, the following people sent correct solutions to the last crossnumber:

Graham Pollard (Woden Valley High, ACT);
Rene Melman (North Sydney Girls' High);
Peter Berry (Macquarie Boys' High);
Garry Helprin (School not given);
M. Harmelin and David Paterson (Sydney Boys' High).
I apologise that these names were not included in Vol. 9 No. 1 – Editor.

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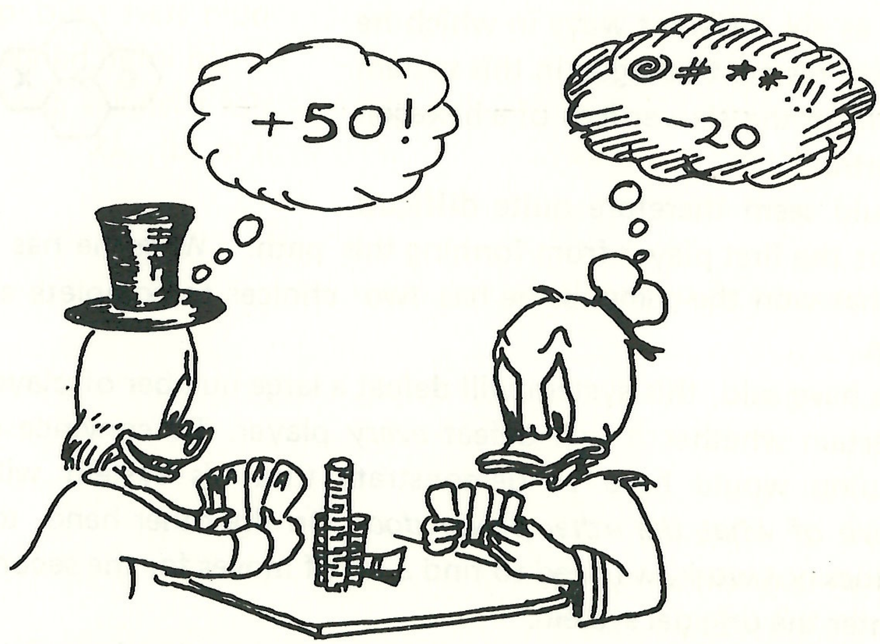

Numerical values must be assigned to each outcome of a mathematical game.