THE FIBONACCI SERIES

Have you ever taken any two numbers, added the second to the first; written this down, added the result to the second number, written the result down, and so on? In the year 1202, Leonardo of Pisa, also known as Fibonacci, did this and the series of numbers is called the Fibonacci sequence or series.

The Fibonacci series uses the number one as a first and second term, and the numbers run like this . . .

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55 \text{ and so on} \ldots \qquad (1)$$

You may wonder what happens if we take another number. Let's call it x. We then get . . .

$$x, x, 2x, 3x, 5x, 13x \ldots$$

The coefficients are identical to series one. Thus if we take any number, we can form a Fibonacci series by multiplying the number by the terms of series one.

Generalising the situation, we can take any two numbers x and y and work out the series:

$$x, y, x+y, x+2y, 2x+3y, 3x+5y, \ldots$$

The original series reappears in the coefficients. So no matter what numbers we take, we can express a series in terms of the original series (1).

This series often occurs in nature. It is found that the number of branches on a tree is given by the Fibonacci series. Another example is as follows:

We select a leaf on the front of the stem of a plant (call the leaf A). Next we tie a piece of cotton to leaf A and pass it around the stem to the next leaf, and so on, leaf to leaf until the next leaf on the front of the stem is reached (call this next leaf in front of the stem B). The cotton will then trace out a spiral pathway connecting leaf A to leaf B.

Starting with leaf A, count the number of complete turns around the stem in passing from A to B (call it p) and count the number of leaves touched by the cotton in passing from A to B (calling this value q). If we measure the ratio p/q, it will be noticed that this ratio is the same for all possible choices of leaf A on the plant. Some plants have the ratio 1/2, some have the ratio 1/3, some 2/5, some 3/8, some 5/13 and so on. If we use F(n) to denote the value of the nth term of the Fibonacci series, then these ratios can be expressed by the formula numbered (2). (See Figure 1.)

$$\frac{F(n)}{F(n+2)} \quad \ldots \qquad (2)$$

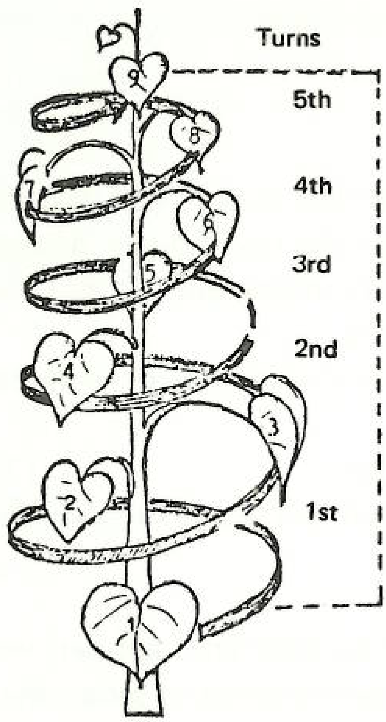

Figure 1. In the example above, there are five complete turns and eight spaces from leaf 1 to leaf 9. The Fibonacci ratio for this plant is 5/8.

[Some actual examples of this in nature are: the elm tree which has ratio 1/2, the beech tree with ratio 1/3, the oak and apricot with ratio 2/5, the poplar and pear with ratio 3/8, and the almond with ratio 5/13 – Ed.]

Another appearance of the Fibonacci series is in the multiplication of two numbers. Let's call them x and y. Continued multiplication by the previous term will give the following series

$$x \quad y \quad xy \quad xy^2 \quad x^2y^3 \quad x^3y^5 \ldots$$

Represented in a table with the term in the left column, the x exponent in the middle column, and the y exponent in the third column, we get the following

Term	x Exponent	y Exponent
x	1	0
y	0	1
xy	1	1
xy^2	1	2
x^2y^2	2	3
x^3y^5	3	5

Reading down the Exponent columns, the numbers follow a Fibonacci sequence. This is not so strange as we are really adding indices to obtain these numbers. We can again form another formula: If the term number is n then the x exponent is F(n–2) and the exponent of y is F(n–1). (Note, however that the formula doesn't work for the first two terms.) We can then calculate the value of the n'th term. This is expressed in the third formula:

$$n\text{'th term} = x^{F(n-2)} \cdot y^{F(n-1)} \ldots \qquad (3)$$

Example: the tenth term $= x^{21}y^{34}$.

If you experiment with numbers, and nature, you will find many applications of this interesting series.

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Editor's Note: Another good article from a reader. If you would like to write an article and can't think of a topic, why not do a follow-up one to this? To get you started, consider the following "continued fractions":

$$1,\ 1+1=2,\ 1+\frac{1}{1+1}=\frac{3}{2},\ 1+\frac{1}{1+\frac{1}{1+1}}=5/3, \text{ and so on.}$$

Note the occurrence of the Fibonacci numbers. Why do they appear? Also what eventually happens to this fraction? Try calculating the ratios for yourself and try to guess. Some reader might even like to calculate mathematically the limit of this sequence.

A useful book on this subject is "Introduction to Geometry" by H.S.M. Coxeter. Ask your school librarian for a copy.

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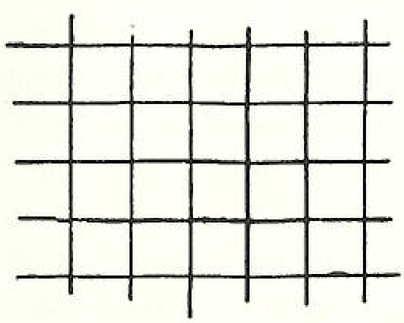
TESSELLATIONS

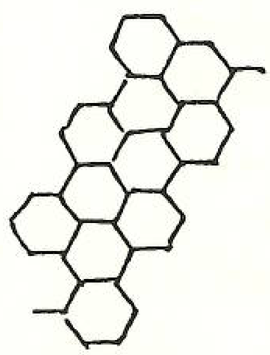
A tessellation is formed when a shape or shapes are repeated so that they would eventually cover the entire plane. Tessellations are sometimes referred to as 'tiling patterns'.

The regular tessellations are formed by repetition of a regular polygon. There are only three of these regular tessellations and they are shown below.

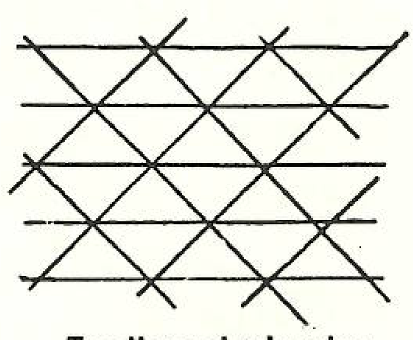
The semi-regular tessellations are formed by regular polygons, so that the polygons surrounding any vertex are identical with those surrounding any other vertex. There exists only eight semi-regular tessellations and you will find these scattered throughout this and subsequent editions of Parabola.



Squares



Hexagons



Equilateral triangles