YOUR LETTERS

Dear Sir,

Soon after writing my previous letter to you, I noticed the error I had made in connection with numbers 7 and 9. I have now been able to make the generalisation that if one uses the method I described in my previous letter, and using an initial digit i, then the resulting number is equal to 1 more than the reciprocal of 1 less than ten times the original digit i, i.e. the resultant number from an initial digit i equal to $1 + \frac{1}{10i-1}$. This generalisation can be carried further. If the numeration base to which one is working is b, then the number resulting from the method in base b is equal to $1 + \frac{1}{bi-1}$. I have been able to show this by using the APL programming language on an IBM 1130 computer, though at present I have no formal proof.

Mark Diamond,
Hollywood Senior High, W.A.

Dear Sir,

In replying to Mark Diamond's query in Vol. 9 No. 2 on why dividing by five is 1 + 1/49, I think I have an answer.

To divide by five, simply multiply by two and divide by ten, thus

$$\begin{array}{r|l} & 1 + 2.10^{-2} + 4.10^{-4} + \ldots + 2n.10^{-2n} \\ \hline 5 & 5 + 1.10^{-1} + 2.10^{-3} + \ldots + n.10^{-2n+1} \end{array}$$

The quotient is the sum of a geometric progression.

Let a = the first element, r = the ratio.

$$\text{Sum} = \frac{a(1-r^n)}{1-r} = \frac{1-(1/50)^n}{49/50} = \frac{50-50(1/50)^n}{49} = \frac{50-(1/50)^{n-1}}{49}$$

As $n \to \infty$, $(1/50)^{n-1} \to 0$.

Thus Sum = 50/49

= 1 + 1/49.

As for the other two, I can't figure them out.

Peter Hehir,
St. Ignatius College, N.S.W.

Can anyone do the other cases? — Ed.

Dear Sir,

Referring to the article by Mark Diamond: Those integers which, when divided by their first digit are solved simply by moving that digit to the other end of the integer have a couple more interesting features.

Firstly the integer for 6: 10169 49152 54237 28813 55932 20338 98305 08474 57627 118644 067796 6.

Mark left the first 6 at the wrong end for it makes another double at the other end. Taking the first double 72881 it can be seen that 7 + 2 equals 8 + 1. Similarly with 13559, we have 1 + 3 = the units digit of 5 + 9, and with 93220 where 2 + 0 = the units digit of 9 + 3. This works for the lot except the groupings 20338 where 2 + 0 = 1 + the units digit of 3 + 8, and 97661 (taking the first digit from the beginning and putting it after the last 6) where 1 + the units digit of 7 + 9 = 6 + 1. This fault in the system could perhaps be blamed on the proximity of the preceding double in each case.

Treating the integer for 5: 10204 0816 3265 30612 24489 79591 83673 46938 7755, it is rather obvious that it is a type of progression 1, 2, 4, 8, 16, 32, 64 etc. each two places further along than the one before and added together when overlapping occurs. The reason for this happening is rather obvious.

The most fascinating discovery was for 9: 10112 35955 05617 97752 80898 87640 44938 20224 719. Taking the digits from left to right it is seen as a worn out second rate Fibonacci sequence. [See Charles Cave's article in this issue — Ed.]

It will be claimed that imperfections emerge but these all fit into a pattern which I think is easily explained though I have not completed doing so yet.

Another interesting point is clearly seen if I lay out each double digit and the two previous digits.

1011
5955
7977
8988
4044
2022

If the repeated digit $\geqslant 5$, then preceding digit is 9, if the repeated digit < 5, then preceding digit is 0.

The reasons for this are involved with the nature of the Fibonacci sequence.

There are several other interesting features of these peculiar types of numbers which I have no room to write about but I'm sure that other readers can find equally interesting aspects.

Chris Sparks,
Newington College

Dear Sir,

In your answer to "problem squares" (Vol. 9 No. 2), you state that $m = n^2$ is impossible. I believe you have not considered the cases similar to

$$\underbrace{\underbrace{000 \ldots 000}_{97 \text{ zeros}} \; 81}_{m} \; \underbrace{9}_{n}$$

Also in your article "Geometry — all done with mirrors" you have used $S_l S_m$ to stand for reflection in l followed by reflection in m, while at school we learnt that one works from right to left, in other words reflection in m before reflection in l. This is important since

$$S_l S_m \neq S_m S_l.$$

A. Fekete,
Sydney Grammar

Editor's comments: 1. Thank you for pointing out the mistake — the question should have read "digits from 1 to 9" (someone might like to answer the question taking notice of Alan's correction).

2. Some mathematicians write it one way and some the other. Of course they must be consistent for, as Alan says, the commutative law is not true!

Dear Sir,

Recently in Maths class (4th Form) dealing with the Factor Theorem, a question was asked "Find a factor of $x^{27} + 1$", meaning that -1 was an obvious root, hence $x + 1$ was a factor. However, looking into this more closely, the expression $x^{27} + 1$ can be factorized beautifully to prove this result, simply by knowing $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ i.e.

$$
\begin{aligned}
x^{27} &= (x^9)^3 + (1)^3 \\
&= (x^9 + 1)((x^9)^2 - x^9 + 1) \\
&= ((x^3)^3 + (1)^3)(x^{18} - x^9 + 1) \\
&= (x^3 + 1)(x^6 - x^3 + 1)(x^{18} - x^9 + 1) \\
&= (x + 1)(x^2 - x + 1)(x^6 - x^3 + 1)(x^{18} - x^9 + 1).
\end{aligned}
$$

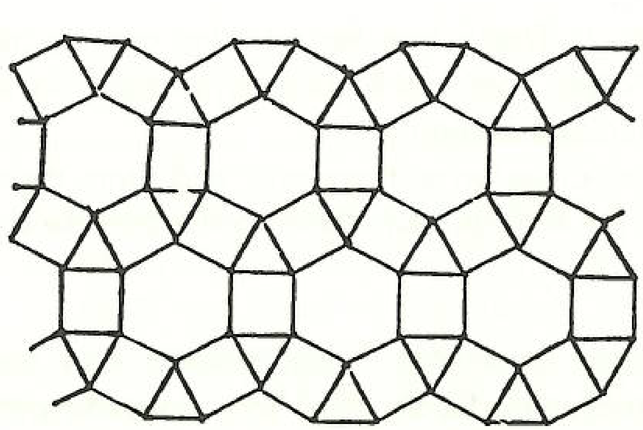
John Burnett,
James Ruse Agricultural High

Dear Sir,

I am enclosing with this letter an article that I wrote myself about the Fibonacci series, which should interest readers. I entered in the recent 1973 Mathematics Competition in the Junior Division, and I appreciate the availability of the solutions so soon after the examination. Your magazine is most interesting and educational, this year's series being my first to read.

Charles M. Cave,
Knox Grammar School

The article is on page 7. — Ed.



Tessellation