EXAMINER'S COMMENTS

SCHOOL MATHEMATICS COMPETITION

Junior Division

1. Most candidates did not understand the question and so ruled themselves out of consideration. A segment here means a line segment; i.e.a definite part of a straight line. Fortunately most of those who did understand the question saw that they had to consider whether all 3 conditions, $p + q > r$, $r + q > p$, $p + r > q$ held.

2. Most solvers correctly obtained (2,1,1) and (–2,–1,–1) as solution sets but only one competitor got any part of (b) as a solution.

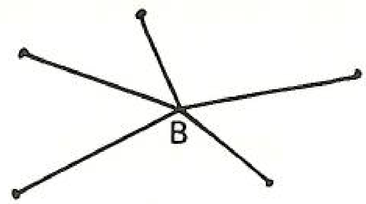
3.(i) There was a fairly good rate of success on this part although there were too many candidates who failed to show how they reached the correct answer. Just listing all 20 triangles would have done fine.

3.(ii) It was understandable that many candidates did not know or had forgotten the meaning of the word 'centroid'. However, all candidates are advised that they may bring any books they like to the examination and can consult them during the examination, so that it seems unfortunate that they didn't presumably do so.

4.(i) Some candidates proved this neatly by showing that the squared integers lie on the curve $y = x^2$, whereas numbers in an arithmetic progression must lie on a straight line which necessarily cannot meet the curve in more than two points.

4.(ii) Some attempted this by using the triangular numbers as to divide the integers into 2 disjoint sets. (A triangular number is of the form $\frac{1}{2}(n)(n + 1)$ and is equal to the sum $1 + 2 + 3 + \ldots + n$). It was ingenious and was awarded some marks, but it doesn't work out.

5. Some candidates saw that A and C were, so to speak, in the same position; whatever was true about C was also true about A. Fewer saw that this in no way restricted the number of members whom B knew. Pictorially if each member is represented by a point we can have single points (no problem); 2 points joined by a line, denoting knowing each other (no problem either); or a star with a B as nucleus joined by rays to any number of other members as below:—



All we have to do is to put the B in one room and all the rest in another. Of course, there may be any number (or none) of single points, pairs and stars and we can attend to each separate one in turn.

This was explained very clearly by Colin Jay of North Sydney Boys High School.

Senior Division

Dear Sir,

I am writing to correct an error in your solution to Problem 1 appearing in Vol. 9 No. 2. You give the solutions for all values of a with $a > 0$, $a \neq 1$. But I shall show that if $a < 1$, there are in fact no solutions.

As you have shown, we must have $t > 0$ and then

$$(|t+1| + |t-1|)/2\sqrt{t} = a.$$

Now consider $f(t) = (|t+1| + |t-1|)2\sqrt{t}$ (for $t > 0$). For $0 < t < 1$, $f(t) = \sqrt{t}$ is decreasing and for $t > 1$, $f(t) = \sqrt{t}$ is increasing.

So the min. value of f(t) is 1, occurring at t = 1. So, to solve

$$f(t) = a,$$

we must have $a \geqslant 1$. But we know $a \neq 1$, so $a > 1$. If $a < 1$, there are no solutions for t. If $a > 1$, there are two solutions $t = a^2$ or $1/a^2$. This completes the solution.

Yours sincerely,

Michael D. Hirschhorn

Mr Hirschhorn is a former student of the University of NSW and is now teaching at the King's School, Parramatta. — Ed.

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