# PROBLEM SECTION

*Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by January 31, 1973, will be published in the next issue Vol. 10 No. 1, 1974. Send all solutions to the Editor (address on inside front cover).*

The problems are classified as Junior, Intermediate or Open. Only students in the first two years of secondary education during 1973 are eligible to submit answers to the Junior problems, and only those in the first four years during 1973 may submit answers to the Intermediate problems. Anyone may submit solutions of problems in the Open section.

## Junior

**J221** Find a 2-digit number AB such that $(AB)^2-(BA)^2$ is a perfect square.

**J222** You can see easily that $3^2 + 4^2 = 5^2$. Prove that there are no 3 consecutive integers such that the cube of the largest equals the sum of the cubes of the others.

## Intermediate

**I223** I think of a whole number x. I cube it. I add the digits of the cube. If I obtain the number I first thought of, find all possible values of x.

**I224** Calculate the following sums:

(a) $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{(n-1).n}$

(b) $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{(n-2)(n-3).n}$

(c) $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\ldots+\frac{1}{(n-3).(n-2).(n-1).n}$.

**I225** If a + b + c = 0, simplify

$$\left(\frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c}\right)\left(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}\right).$$

## Open

**O226** The houses on the same side of the street as Tom's house are numbered 1, 3, 5, . . . (with no odd number left out). The sum of the house numbers from Tom's to the end of the street is the same in both directions. If his house has a 3-digit number, what is it?

**O227** Prove that for all positive integers, n,

$$\frac{\sqrt{2}}{2.\sqrt{(2n)}} \leqslant \frac{1.3.5.\ldots.(2n-1)}{2.4.6.\ldots 2n} < \frac{\sqrt{3}}{2.\sqrt{(2n)}}.$$

**O228** The radius of the inscribed circle of a triangle is 4 and the segments into which one side is divided by the point of contact are 6 and 8 units long. Find the lengths of the other two sides.

**O229** Two identical circular cylinders with unit radius have axes which intersect at right angles. Find the volume of the region inside both cylinders.

**O230** A game for two players involves a heap of matches. The first player, A, may pick up any positive number of matches, so long as at least one is left. Thereafter the players move alternately, a move consisting in picking up any positive number of matches not exceeding twice the number picked up by the opponent's move just completed. For example, if at some stage A picks up 3 matches, B may pick up 1, 2, 3, 4, 5, or 6 matches for his next play. The winner is the player who picks up the last match. If there are initially 60 matches in the heap, the first player can force a win. How?

## SOLUTIONS

**Solutions to Problems 211–220 in Vol. 9 No. 2.**

### Junior

**J211** The numbers 31,767 and 34,924, when divided by a certain 3 digit divisor, leave the same remainder, also a 3-digit number. Find the remainder.

**Answer:** The difference of the two numbers must be a multiple of the divisor, since the remainders are equal. That is, the divisor is a 3-digit factor of 34,924–31,767 (which = 3,157 = 7 x 11 x 41) so must be either 7 x 41 or 11 x