PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by May 1, 1974, will be published in the next issue Vol. 10 No. 2, 1974. Send all solutions to the Problem Editor (address on inside front cover).

The problems are classified as Junior, Intermediate or Open. Only students in the first two years of secondary education during 1974 are eligible to submit answers to the Junior problems, and only those in the first four years during 1974 may submit answers to the Intermediate problems. Anyone may submit solutions of problems in the Open section.

Junior

J231 *(submitted by J. Pike)* A man goes to an auction with \$100 and buys exactly 100 animals. The bulls sell at \$5 each, the sheep sell at \$1 each and the hens sell at 5 cents each. If he spends the whole \$100 and buys some of each of the above three animals, how many of each does he buy? *(Maybe you can send in a similar problem? – Editor)*

J232 In the following calculation, all but one digit is indicipherable. Replace each of the dots by the digit that should have been there.

$$
\begin{array}{cccccccc}
 & & & & & \bullet & \bullet & \bullet \\
 & & & & \times & \bullet & \bullet & \bullet \\
\hline
 & & & & \bullet & \bullet & \bullet & \bullet \\
 & & & \bullet & \bullet & \bullet & \bullet & \\
 & & \bullet & \bullet & \bullet & \bullet & & \\
\hline
 & & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
 & + & & & 1 & \bullet & \bullet & \bullet \\
\hline
 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
\end{array}
$$

Intermediate

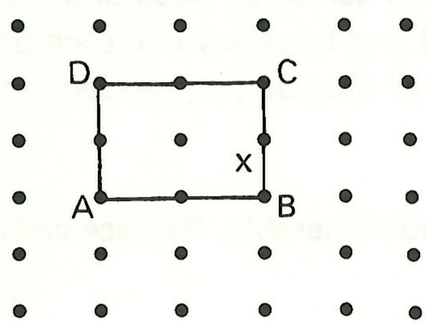
I233 The following are n ($\geqslant 4$) simultaneous equations in the pronumerals $x_1, x_2, x_3, \ldots x_n$.

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 &= 0 \\
x_2 + x_3 + x_4 + x_5 &= 0 \\
x_3 + x_4 + x_5 + x_6 &= 0 \\
&\;\;\vdots \\
x_{n-3} + x_{n-2} + x_{n-1} + x_n &= 0 \\
x_{n-2} + x_{n-1} + x_n + x_1 &= 0 \\
x_{n-1} + x_n + x_1 + x_2 &= 0 \\
x_n + x_1 + x_2 + x_3 &= 0
\end{aligned}
$$

For what values of n is the solution unique? When it is not unique, find the most general solution.

I234 A man views the Pentagon in Washington through binoculars from a large distance away. What is the probability that he can see three sides?

I235 X is a point at the centre of a square array of dots with 2n dots on each side.



(In the diagram shown n = 3). The diagram shows a square, ABCD, whose sides lie on the rows or columns of dots, and which encloses X.

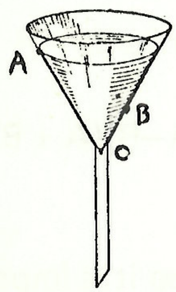
(i) How many such squares may be drawn if n = 1?, 2?, 3?, . . . k?

(ii) How many rectangles enclosing X may be drawn having sides on the rows or columns of dots?

Senior

O236 When Jim Brown came home he asked his wife about the new neighbours. "Three children, including one adopted," she said. Jim wanted to know how old they were. "Work it out for yourself," she said. The product of their ages in *months* is 28,730 and the sum of their ages is exactly equal to our son's age." Jim heaved a sigh, but meekly sat down with pen and paper. After checking his calculations half a dozen times, he pointed out accusingly that he didn't have enough information. "Oh, I'm sorry I forgot to tell you that none of them is as old as our daughter." "Alright, now I know their ages," said Jim. How old are the *Brown's* children?

O237 The diagram shows a filter funnel lined with a piece of filter paper in the usual fashion, and held with its axis vertical. (The circular filter paper is folded twice, resulting in a quadrant shape. This is then opened out into a cone which fits snugly into the funnel. If you have never done any chemistry ask someone from the science class to show you.)



An ant is situated at the point A on the edge of the filter paper (whose radius OA is 10 cms) and a spot of honey is at B, on the opposite side of the funnel and 5 cms from O. The ant walks along the shortest path to the honey. How far is the ant from O when its path is horizontal? Also what is the semi-vertical angle of a conical filter funnel? (i.e. the angle between OA and the vertical axis?)

O238 Find all solutions of the simultaneous equations

$$ab + cd = 25$$
$$ac + bd = 25$$
$$ad + bc = 25$$
$$a + b + c + d = 15$$

O239 Prove that $\cos \frac{\pi}{2^{n+1}} = \frac{\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}}}{2}$

where there are n square root signs.

O240 All entrants for a certain school chess tournament were from form IV or form V. There were nine times as many from the lower form, but between them they only scored 4 times as many points as the entrants from form V. The tournament was conducted on the Round Robin system (i.e. each entrant played all the others once, scoring 1 for a win, ½ for a draw and 0 for a loss). What was the winner's score?