SOLUTIONS

Solutions to Problems 221–230 in Vol. 9 No. 3

Junior

J221 Find a 2-digit number AB such that $(AB)^2-(BA)^2$ is a perfect square.

Answer: The 2-digit number AB is equal to $10A + B$. Since $x^2 - y^2 = (x-y)(x+y)$, we see that

$$(10A + B)^2 - (10B + A)^2 = (9A-9B)(11A + 11B) = 3^2.11(A-B)(A+B).$$

It is clear that this is a perfect square if and only if

$$(A-B)(A+B) = 11k^2 \quad \text{where k is an integer} \qquad (1)$$

Remembering that A and B are digits, A–B is less than 10 and so it is impossible that A–B is a multiple of 11. (We are here ignoring the trivial solutions in which A = B.) Hence A + B must be a multiple of 11 not exceeding 18 (since A and B are digits) i.e. A + B = 11.

From (1) we now obtain $A-B = k^2$.

Since A–B is less than 10, there are only 3 values of k^2 to be tried, viz. 1, 4, and 9. Only the first of these yields a solution in positive integers less than 10 viz. A = 6, B = 5. Thus the only answer is given by $(65)^2-(56) = 9.121 = (33)^2$.

Successful Solvers: J. Archibald, R. Borg, G. Cleary, D. Crawford, P. Hatzi, T. Hatziandreou, R. Hawkins, S. Howdin, G. Longbottom, P. Lorizzo, M. Moy, G. Sherriff, W. Williams, (all from South Sydney Boys' High); M. Reynolds (Marist Brothers, Pagewood).

J222 You can see easily that $3^2 + 4^2 = 5^2$. Prove that there are no 3 consecutive integers such that the cube of the largest equals the sum of the cubes of the others.

Answer: Suppose on the contrary that there are 3 consecutive integers n–1, n, n + 1 such that $(n-1)^3 + n^3 = (n+1)^3$. Simplifying yields $n^3 - 6n^2 - 2 = 0$ or

$$n^2(n-6) = 2.$$

There are several ways of seeing that no integer n satisfies this equation. For example, if n is odd the left hand side is odd and so cannot equal 2, whilst if n is even the left hand side is divisible by 4 and so cannot equal 2. Or, again, n^2 must clearly be a factor of 2. The only possibilities are $n = \pm 1$, neither of which is a solution.

1223 I think of a whole number x. I cube it. I add the digits of the cube. If I obtain the number I first thought of, find all possible values of x.

Answer: If x has k digits (so that $10^{k-1} < x$) then x^3 has at most 3k digits, each digit being at most 9, whose sum x cannot exceed 3k.9. Hence $10^{k-1} < 27k$. It follows easily by trial that k = 1 or 2. i.e. x has at most 2 digits. Its cube then has at most 6 digits and so $x \leqslant 54$. In fact, since the first digit of $54^3, 53^3, \ldots 47^3$ is a 1 not a 9, the largest possible sum of the 6 digits is 46. The possibilities x = 46, 45 or 44 can also be discarded by observing that the last digits of the cubes are 6, 5 and 4 respectively, so the sum of digits is at most 43. Thus $x \leqslant 43$.

Next we use the well known fact that, when divided by 9, any whole number leaves the same remainder as the sum of its digits does. It follows that x and x^3 must both leave the same remainder on division by 9. If we write x = 9q + r with r = 0,1,2, . . . 7 or 8, then $x^3 = 729q^3 + 243q^2r + 27qr^2 + r^3$. Since x is the sum of the digits of x^3 r must be the remainder when r^3 is divided by 9. Trying all the above values for r, we see that r = 0, 1 or 8 and so (since $x \leqslant 43$) x = 1, 8, 9, 10, 17, 18, 19, 26, 27, 28, 35, 36, 37.

Trying these in turn we find that the ones which satisfy the conditions are: x = 1, 8, 17, 18, 26 or 27. (Their cubes are, in order, 1, 512, 4193, 5832, 17576, and 19,683.)

Successful Solvers: C. Bonkowsky (Kogarah High), J. Burnett (James Ruse Ag. High), D. Crocker (Sydney Boys' High) – partly, M. Diamond (Hollywood Senior High W.A.), J. Holten (East Hills Boys' High). Also P. Anderson, J. Archibald, R. Borg, G. Burgess, G. Cleary, D. Crawford, P. Hatzi, R. Hawkins, S. Howdin, S. Lawrence, P. Lorizzo, R. McLachlan, M. Moy, M. Pandoleon, P. Stimitsiotis, B. Talbot, S. Walters, R. Williamson (all from South Sydney Boys' High).

1224 Calculate the following sums:

(a) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{(n-1).n}$

(b) $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{(n-2)(n-1).n}$

(c) $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \ldots + \frac{1}{(n-3).(n-2).(n-1).n}$.

Answer: (a) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} \dots + \frac{1}{(n-1)n} =$

$$(\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) \dots + \dots + (\frac{1}{(n-1)} - \frac{1}{n})$$

$$= 1 - \frac{1}{n}.$$

(b) $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots \frac{1}{(n-2)(n-1)n}$

$$= \frac{1}{2}(\frac{1}{1.2} - \frac{1}{2.3}) + \frac{1}{2}(\frac{1}{2.3} - \frac{1}{3.4}) + \frac{1}{2}(\frac{1}{3.4} - \frac{1}{4.5}) \dots$$

$$+ \frac{1}{2}(\frac{1}{(n-2)(n-1)} - \frac{1}{(n-1)n})$$

$$= \frac{1}{2}(\frac{1}{1.2} - \frac{1}{(n-1)n})$$

(c) $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \dots + \frac{1}{(n-3)(n-2)(n-1)n}$

$$= \frac{1}{3}(\frac{1}{1.2.3} - \frac{1}{2.3.4}) + \frac{1}{3}(\frac{1}{2.3.4} - \frac{1}{3.4.5}) + \dots$$

$$\frac{1}{3}(\frac{1}{(n-3)(n-2)(n-1)} - \frac{1}{(n-2)(n-1)n})$$

$$= \frac{1}{3}(\frac{1}{1.2.3} - \frac{1}{(n-2)(n-1)n}).$$

Successful Solvers: J. Burnett (James Ruse Ag. High), C. Sparks (Newington), Part (a) only — D. Crocker (Sydney Boys' High), M. Diamond (Hollywood Senior High W.A.), S. Hood (James Ruse Ag. High).

1225 If a + b + c = 0, simplify

$$(\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c})(\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b}).$$

Answer: The expression simplifies down to the constant value 9 (assuming no two of a, b, c, are equal, when it is undefined). This can be straightforwardly, if somewhat tediously, shown by replacing c by −a−b everywhere. When each factor is brought to a common denominator, the numerator of the first is equal to the denominator of the second (or its negative) and the numerator of the second is equal to 9 times (or −9 times) the denominator of the first.

Alternatively, make use of the identity

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

which, since $a + b + c = 0$, yields

$$a^3 + b^3 + c^3 = 3abc. \qquad (1)$$

Also observe that

$$a^2(b + c) + b^2(c + a) + c^2(a + b) = -a^3 - b^3 - c^3 = -3abc. \qquad (2)$$

The first factor is A/B where $B = abc$ and

$A = bc(b-c) + ca(c-a) + ab(a-b) = b^2c - bc^2 + c^2a - ca^2 + a^2b - ab^2$

$\quad = -(a-b)(b-c)(c-a)$

The second factor is C/D where $D = (b-c)(c-a)(a-b) = -A$ and

$C = a(c-a)(a-b) + b(a-b)(b-c) + c(b-c)(c-a)$

$\quad = -a^3 - b^3 - c^3 + a^2(b + c) + b^2(c + a) + c^2(a + b) - 3abc$

$\quad = -3abc - 3abc - 3abc$ (using equations (1) and (2))

$\quad = -9abc$

$\quad = -9B$

Multiplying the two factors gives the stated result of 9.

Successful Solvers: J. Burnett (James Ruse Ag. High), D. Crocker (Sydney Boys' High), S. Hood (James Ruse Ag. High), C. Sparks (Newington).

Open

O226 The houses on the same side of the street as Tom's house are numbered 1, 3, 5, . . . (with no odd number left out). The sum of the house numbers from Tom's to the end of the street is the same in both directions. If his house has a 3-digit number, what is it?

Answer: Let Tom's house number be x and let the largest house number on his side of the street be $2y-1$. Now $1 + 3 + 5 + \ldots + x = (\frac{x+1}{2})^2$ and $x + (x + 2) + \ldots (2y-1) = y^2 - (\frac{x-1}{2})^2$.

Equating these yields

$$2y^2 - x^2 = 1. \qquad (1)$$

From the identity $2(2x_1 + 3y_1)^2 - (3x_1 + 4y_1)^2 = 2{y_1}^2 - {x_1}^2$ we observe immediately that if $x = x_1$, $y = y_1$ is a solution of (1), so is

$$x = 3x_1 + 4y_1,\ y = 2x_1 + 3y_1 \qquad (2)$$

Starting with the obvious solution $x = 1$, $y = 1$, successive applications of (2) yields the following set of solutions of (1) in pairs of positive integers

$$(x,y) = (1,1);\ (7,5);\ (41,29);\ (239,169);\ (1393,985) \text{ and so on.} \qquad (3)$$

We have found one solution having a 3 digit value of x, viz. $x = 239$. We are now confident that Tom's house number is 239 and the largest house number on his side of the street (2y–1) is 337.

To show that this is the only solution of the puzzle, we need to prove that all solutions of (1) in pairs of positive integers are contained in the list (3).

Suppose on the contrary, that there is a solution (X,Y) of (1) between the nth and (n+1)th pairs in the list (3), i.e. $x_n < X < x_{n+1}$, $y_n < Y < y_{n+1}$.

We ask now "Is there a smaller solution (X_1,Y_1) which upon applying the formula (2) yields (X,Y)?" i.e. we attempt to solve $3X_1 + 4Y_1 = X$, $2X_1 + 3Y_1 = Y$ for X_1 and Y_1.

This gives easily $X_1 = 3X-4Y$, $Y_1 = -2X + 3Y$. It is clear that this solution of (1) in positive integers lies between (x_{n-1}, y_{n-1}) and (x_n,y_n) (since application of (2) to (X_1,Y_1) yields a solution in the next interval). It follows that if there is any solution of (1) not included already in the list (3), there must be such a solution between (1,1) and (7,5). One has only to try x = 2,3,4,5 and 6 in turn to rule out this possibiilty.

Note: The above involves solving the equation (1) – which is known as Pell's equation – for integers x,y. The following is an alternative answer making use of Pythagorean triples (see Vol. 7 No. 3, or ask your teacher to work them out for you).

Let Tom's house number be 2m + 1 and let the largest house number on his side of the street be 2n + 1.

Now $1 + 3 + 5 + \ldots + (2m + 1) = (m + 1)^2$ and $(2m + 1) + (2m + 3) + \ldots + (2n + 1) = (n + 1)^2 - m^2$.

Equating these yields $m^2 + (m + 1)^2 = (n + 1)^2$ (1)

Thus m, m+1, n+1 form a Pythagorean triple and so we can find an expression for them. If m is even, there are positive integers a,b such that

$$m = 2ab, \quad m + 1 = a^2 - b^2 \qquad (2)$$

where a–b is odd, and so we can write a–b = 2z + 1 for positive integer z.

Substituting in (2), we get m = 2b(b + 2z + 1) and (2z + 1)(2b + 2z + 1) = m + 1 = 2b(b + 2z + 1) + 1.

Simplifying,

$$2z(z + 1) = b^2 \qquad (3)$$

But b < a and so $8z(z + 1) = 4b^2 < 4ab = 2m < 1000$ (since Tom's number has 3 digits) i.e. z < 11. The only numbers less than 11 which can replace z in equation (3) are z = 1 and z = 8, giving b = 2, a = 5, m = 20, 2m + 1 = 41 and b = 2, a = 29, m = 696, 2m + 1 = 1393, the first being too small and the second being too large for Tom's number.

If m is odd, there are positive integers a,b such that

$$m = a^2 - b^2, \quad m + 1 = 2ab \qquad (2)'$$

Doing the same algebra as before, we get

$$z^2 + (z+1)^2 = b^2 \qquad (3)'$$

with $z < 11$ again. Either repeating the argument or checking the numbers less than 11 we get as the only answer $z = 3$, $b = 5$, $a = 12$, $m = 119$, and so Tom's number is 239.

Successful Solvers: J. Burnett (James Ruse Ag. High), D. Crocker (Sydney Boys' High), M. Diamond (Hollywood Senior High W.A.), A. Fekete (Sydney Grammar) – by computer, A. Oliviero (Newington), D. Powers (Fort Street Boys' High), R. Casley (Gosford High).

O227 Prove that for all positive integers, n,

$$\frac{\sqrt{2}}{2.\sqrt{(2n)}} \leqslant \frac{1.3.5.\ldots.(2n-1)}{2.4.6.\ldots.2n} < \frac{\sqrt{3}}{2.\sqrt{(2n)}}.$$

Answer: Set $x = \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}$ where $n \geqslant 2$

Then $\frac{1}{x} = \left(\frac{2.4.6\ldots(2n-2)}{3.5\ldots(2n-1)}\right) 2n = 4n . \frac{1}{2} . \left(\frac{2}{3} . \frac{4}{5} \ldots \frac{2n-2}{2n-1}\right).$

$< 4n . \frac{1}{2} . \frac{3}{4} . \frac{5}{6} \ldots . \frac{2n-1}{2n}$

Since each of the factors inside the round brackets has been slightly increased by adding one to both numerator and denominator.

i.e. $\frac{1}{x} < 4nx$ whence $x^2 > \frac{1}{4n}$ and $x > \frac{\sqrt{2}}{2\sqrt{(2n)}}$

To prove the second inequality

$$\frac{1}{x} = \frac{2}{3} . \left(\frac{4}{5} . \frac{6}{7} \ldots . \frac{2n-2}{2n-1}\right) . 2n > \frac{4}{3} . 2n . \frac{1}{2} \left(\frac{3}{4} . \frac{5}{6} \ldots \frac{2n-3}{2n-2}\right) . \frac{2n-1}{2n}$$

where we have not only decreased each factor within the round brackets but also introduced an extra factor less than one.

Hence $\frac{1}{x} > \frac{4}{3} . 2n . x$, $\quad x^2 < \frac{3}{4.2n}$ and $x < \frac{\sqrt{3}}{2\sqrt{(2n)}}$

If $n = 1$, the above working is meaningless (there are no terms left inside the round brackets) but it is easy to check that

$$\frac{\sqrt{2}}{2.\sqrt{(2.1)}} = \frac{1}{2} = x < \frac{\sqrt{3}}{2.\sqrt{(2.1)}}$$

Successful Solvers: J. Burnett (James Ruse Ag. High), D. Crocker (Sydney Boys' High) — not quite complete, R. Kuhn (Sydney Grammar) — partly, D. Powers (Fort Street Boys' High).

O228 The radius of the inscribed circle of a triangle is 4 and the segments into which one side is divided by the point of contact are 6 and 8 units long. Find the lengths of the other two sides.

Answer: Using the figure $BI = \sqrt{(4^2 + 8^2)} = 4\sqrt{5}$ and so

$\sin B = \sin 2\beta = 2 \sin \beta . \cos \beta$

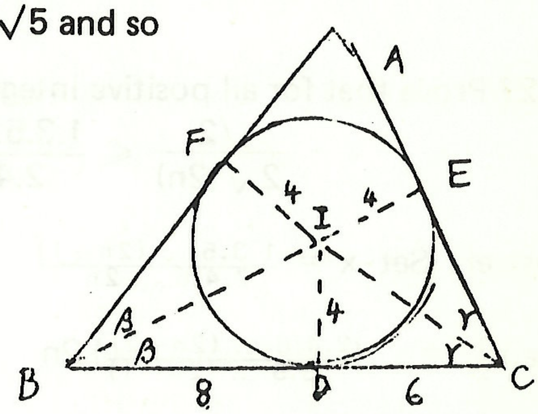
$= 2(4/4\sqrt{5}) \; . \; (8/4\sqrt{5})$

$= 4/5$

Similarly, $\sin C = \sin 2\gamma = 12/13$.

Applying the sine rule to triangle ABC ($b \sin C = c \sin B$)

$$(6 + x).\tfrac{12}{13} = (8 + x).\tfrac{4}{5}$$



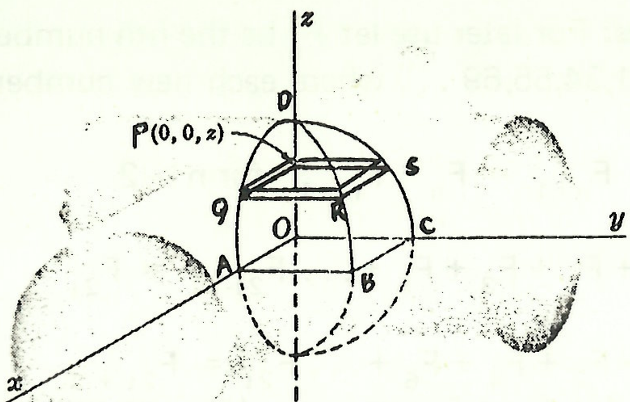
The solution of this is $x = 7$, so that the sides AB and AC are of lengths 15 and 13 respectively.

Successful Solvers: C. Bonkowsky (Kogarah High), J. Burnett (James Ruse Ag. High), D. Crocker (Sydney Boys' High), M. Diamond (Hollywood Senior High W.A.), J. Holten (East Hills Boys' High), R. Kuhn (Sydney Grammar), A. Oliviero (Newington), D. Powers (Fort Street Boys' High), C. Sparks (Newington), R. Casley (Gosford High).

O229 Two identical circular cylinders with unit radius have axes which intersect at right angles. Find the volume of the region inside both cylinders.

Answer: The diagram shows one-eighth of the region. The front face DQABR is part of the surface of one of the cylinders, radius 1, axis Oy. This cylinder cuts the xOz plane in the curve AQD, a quadrant of the unit circle centre O, whose equation in this plane is $x^2 + z^2 = 1$. Similarly, the other cylinder, with axis Ox, cuts the zOy plane in the arc DSC whose equation is $y^2 + z^2 = 1$.

A Plane parallel to xOy cuts this region in a square cross section (e.g.



PQRS). Indeed, if the length of OP is z, the lengths of both PQ and QS are found from the equations obtained to be $\sqrt{(1-z^2)}$. Hence PQRS is a square of area $(1-z^2)$. The volume, δV, of a very thin slice of the region diagrammed is then given by $\delta V \cong (1-z^2)\,\delta z$, where δz is the thickness of the slice. The error in this approximation becomes negligible as δz approaches zero. Hence the total volume V, the sum of all slices if a large number of such planes are drawn, is given by

$$V = \int_0^1 (1-z^2)\,dz = \left[z - \tfrac{1}{3}z^3 \right]_{z=0}^{z=1} = \tfrac{2}{3}$$

The required answer is eight times the volume of the diagrammed region (note that the full region is symmetric about all the planes xOy, yOz, xOz); namely $\frac{16}{3}$.

Successful Solvers: R. Casley (Gosford High), D. Crocker (Sydney Boys' High), M. Diamond (Hollywood Senior High W.A.), D. Powers (Fort Street Boys' High), R. Kuhn (Sydney Grammer) gave an excellent solution, including several more difficult results e.g., the volume common to 3 intersecting cylinders mutually at right angles.

O230 A game for two players involves a heap of matches. The first player, A, may pick up any positive number of matches, so long as at least one is left. Thereafter the players move alternately, a move consisting in picking up any positive number of matches not exceeding twice the number picked up by the opponent's move just completed. For example, if at some stage A picks up 3 matches, B may pick up 1, 2, 3, 4, 5, or 6 matches for his next play. The winner is the player who picks up the last match. If there are initially 60 matches in the heap, the first player can force a win. How?

Answer: (a) Preliminaries: For later use let F_n be the nth number in the Fibonacci sequence 1,2,3,5,8,13,21,34,55,69 . . . where each new number is the sum of the preceding 2, i.e.

$$F_{n+1} = F_n + F_{n-1} \quad \text{for } n > 2 \tag{1}$$

Observe that

$$1 + F_1 + F_3 + F_5 + \ldots F_{2t-1} = F_{2t} \tag{2}$$

and that

$$1 + F_2 + F_4 + F_6 + \ldots F_{2t} = F_{2t+1} \tag{3}$$

To prove (2), note that $1 + F_1 = F_2$; now use (1) repeatedly, replacing the first two terms of the sum by a single term. A similar method obviously suffices to prove (3). It follows immediately that any whole number R less than F_r can be written uniquely as

$$R = F_{r-1} + F_{r-3} + \ldots + F_{r-2s+1} + X \quad \text{where } 1 \leqslant X \leqslant F_{r-2s-1} \ (s = 0,1,\ldots) \tag{4}$$

[For example, take $r = 7$, $F_r = 21$. If $R \leqslant 13$, $R = X$ where $X \leqslant F_6$. If $14 \leqslant R \leqslant 18$, $R = 13 + X = F_6 + X$ where $1 \leqslant X \leqslant 5 = F_4$. If $R = 19$ or 20, $R = 13 + 5 + X$ where $X = 1$ or 2, and $X \leqslant F_2$.]

Finally using (1) we can show that

$$F_n > 2.F_{n-2} \tag{5}$$

(b) Solution of Problem: We claim that if the number of matches N initially in the heap is a Fibonacci number, i.e. $N = F_n$ $(n > 1)$, then B can force a win with correct play. Otherwise A can force a win. This can be proved by induction:— First of all, if N is small, the claim can easily be checked by experiment. Suppose it is true whenever the number of matches is at most F_k for some k, and consider any N in the interval $F_k < N < F_{k+1}$. Set $N = F_k + R$ where $1 \leqslant R \leqslant F_{k-1}$. Using (4), we may divide the pile of matches into smaller healps $H_0, H_1, H_2, \ldots H_s, H_{s+1}$, containing respectively $F_k, F_{k-2}, F_{k-4}, \ldots F_{k-2s}$ and X matches where $X \leqslant F_{k-2s-2}$. A's winning strategy consists in taking all X matches in heap H_{s+1}. Since, in view of (5), B cannot remove all of the heap H_s on his next move, and since H_s contains F_{k-2s} matches, by the induction hypothesis A can so play that on some later move he takes the last match in H_s. Similarly, A can play so as to take the last match of H_{s-1}, then of $H_{s-2}, H_{s-3} \ldots$ and eventually of H_0.

[Of course, it is irrelevant that B does not have to take the matches from any particular smaller heap. A can rearrange them each time before he moves if he so wishes.]

We complete our inductive proof of the above claim by considering the case $N = F_{k+1}$ $(= F_k + F_{k-1})$. We have to show that B can force a win. If A takes a number of matches $Y \geqslant F_{k-1}$, B can win immediately by taking all the others.

However, if $Y < F_{k-1}$ the number of matches left is between F_k and F_{k+1} and B can win by adopting the strategy recommended above for A. [For reasons of space, we will omit the proof that the number X which he must remove never exceeds 2Y. It is not very difficult to show using (2) (or (3)) and (4) above.]

For example, if there are 60 = 55 + 5 matches initially in the heap, A can win by taking 5 matches on his first move. Note that 55 = 34 + 13 + 5 + 2 + 1 and that B cannot choose more than 10 matches on his first move. Suppose, by way of example, he chooses Y matches where $3 \leqslant Y < 8$, leaving 34 + 13 + X matches, where $1 \leqslant X \leqslant 5$. A answers by removing X matches leaving 34 + 13 matches. Similarly by adopting the strategy outlined, A can play so as to capture the last of the next 13 matches leaving 34 matches. If B resists without unnecessarily surrendering by allowing A to go out on the next move, he will at later stages be confronted with a heap of size 21, 13, 8, 5, and finally 3. B can now take either 1 or 2 matches, and then A removes the rest.

Successful Solvers: R. Casley (Gosford High) – correct method but some errors, M. Diamond (Hollywood Senior High W.A.), A. Fekete (Sydney Grammar) – see also his letter in this issue, A. Oliviero (Newington), D. Powers (Fort Street Boys' High) – right idea for an attack on the problem but errors occurred in constructing his sequence of winning plays, R. Kuhn (Sydney Grammar) correctly announced that the game was a win for the first player unless the number of matches was a Fibonacci number. I was unable to follow his explanation as to why this was so.

☆ ☆ ☆

HIAWATHA'S THEOREM (from our first issue)

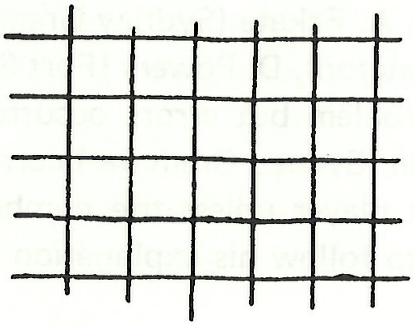
Three Navaho women sit side by side on the ground. The first woman, who is sitting on a goatskin, has a son who weighs 140 pounds. The second woman, who is sitting on a deerskin, has a son who weighs 160 pounds. The third woman, who weighs 300 pounds, is sitting on a hippopotamus skin. What famous geometric theorem does this symbolize? (Answer on page 32)

TESSELLATIONS

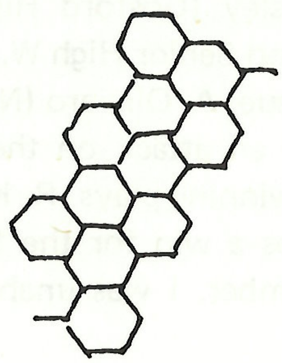
A tessellation is formed when a shape or shapes are repeated so that they would eventually cover the entire plane. Tessellations are sometimes referred to as 'tiling patterns'.

The regular tessellations are formed by repetition of a regular polygon. There are only three of these regular tessellations and they are shown below.

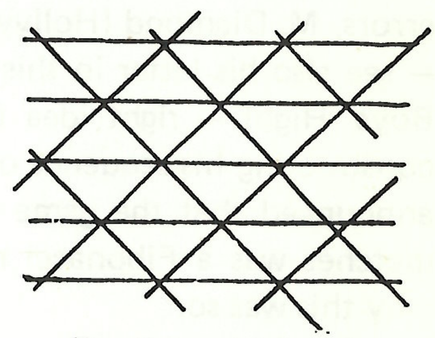
The semi-regular tessellations are formed by regular polygons, so that the polygons surrounding any vertex are identical with those surrounding any other vertex. There exists only eight semi-regular tessellations and you will find these on the following pages and in Parabola Vol. 9 No. 3.



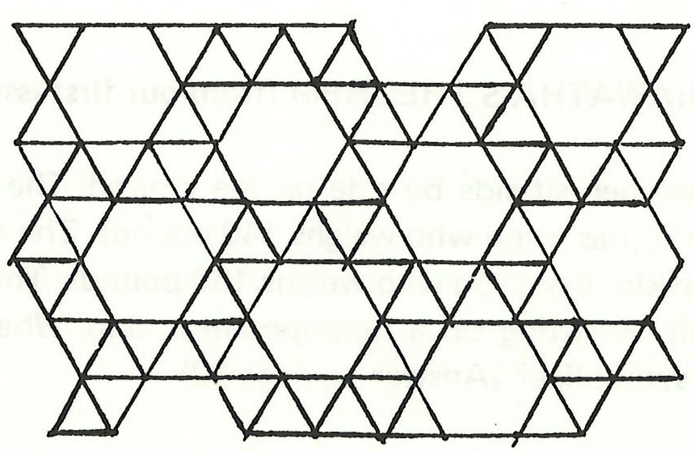
Squares

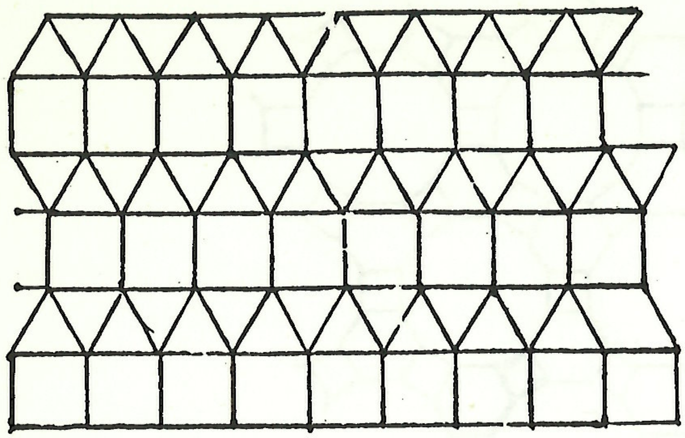


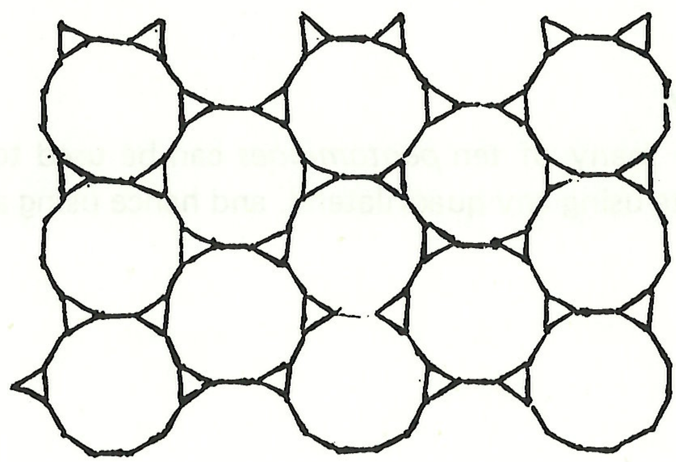
Hexagons



Equilateral triangles

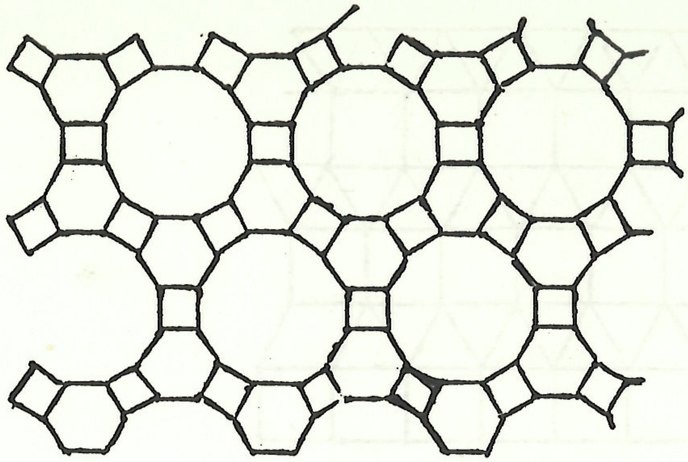






MARKET RESEARCH (from our first issue)

Two men went into a hardware store and enquired the price of certain articles. "Fourpence each," said the shopkeeper. "I'll take seventy seven," said the first man, paying the shopkeeper eightpence. "I'll take one hundred and eight. Here is one shilling," said the second man. What did they buy? (Answer on page 32)



Activity

How many of ten *pentominoes* can be used to tessellate. Prove that you can tessellate using any quadrilateral, and hence using any triangle.

☆ ☆ ☆

ANSWERS

Hiawatha's Theorem: "The squaw on the hippopotamus is equal to the sons of the squaws on the other two hides." (Pythagoras)

Market Research: The men bought numbers for their houses, 77 requiring two figures at fourpence each, and 108 requiring three figures at fourpence each.