MATHEMATICAL GAMES

GLADIATORS

This game is a contest between two gladiators in an arena consisting of a 20 x 20 grid (see Figure 1). The two contestants are each armed with a net which has three corners, the gladiator holding one corner and a heavy lead weight being attached to each of the other two corners. Each one is trying to throw his net over the other gladiator to stop him from moving about and so make it possible to kill him.

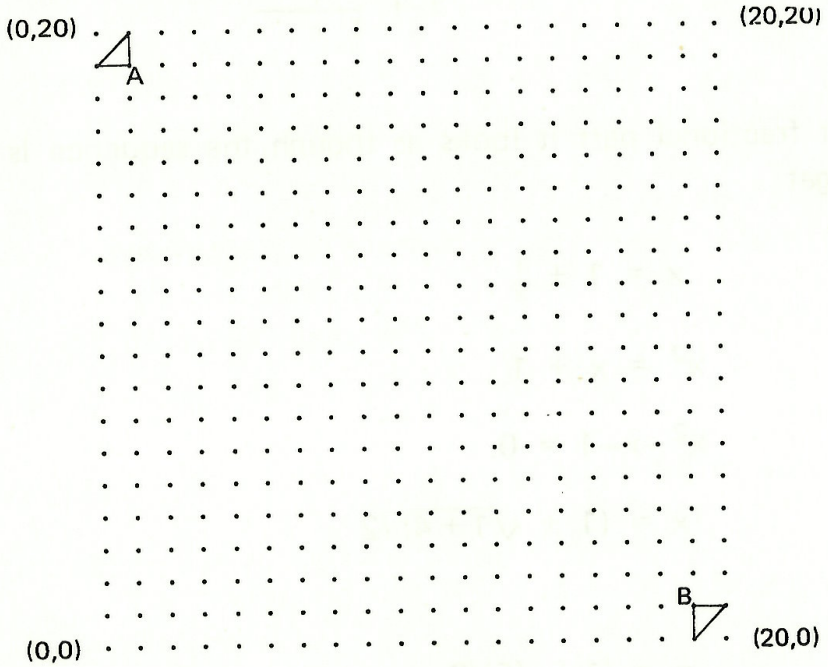

Figure 1.

The contest begins as in Figure 1. Gladiator A is at the point (1,19) with the other corners of his net at the points (1,20) and (0,19), and gladiator B is at the point (19,1) with the other corners of his net at (20,1) and (19,0). A drachma is thrown to decide who has the first throw.

When it is his turn, a gladiator throws his net by choosing a matrix with positive or negative digits or 0 as its elements, and applying that matrix to the co-ordinates of the corners of his net. As in the case of matrix codes (see article on page 2), as many multiples of 20 are subtracted (or added) as possible so as to make the numbers between 0 and 20. In the case when 0 or 20 is thus obtained, the gladiator may choose either of these numbers. A capture is made if the opponent is now completely inside the net in its new position.

For example, suppose gladiator A has had first throw and is at the point (11,9), and gladiator B chooses the matrix

$$\begin{bmatrix} 3 & -9 \\ -9 & 3 \end{bmatrix}.$$

Then $\begin{bmatrix} 3 & -9 \\ -9 & 3 \end{bmatrix} \begin{bmatrix} 19 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\times19 - 9\times1 \\ -9\times19 + 3\times1 \end{bmatrix} = \begin{bmatrix} 48 \\ -108 \end{bmatrix} \rightarrow \begin{bmatrix} 8 \\ 12 \end{bmatrix}$

and so gladiator B moves to the point (8,12). Similarly, the other corners of his net move to

$$\begin{bmatrix} 3 & -9 \\ -9 & 3 \end{bmatrix} \begin{bmatrix} 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 51 \\ -177 \end{bmatrix} \rightarrow \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

and

$$\begin{bmatrix} 3 & -9 \\ -9 & 3 \end{bmatrix} \begin{bmatrix} 19 \\ 0 \end{bmatrix} = \begin{bmatrix} 57 \\ -171 \end{bmatrix} \rightarrow \begin{bmatrix} 17 \\ 9 \end{bmatrix}$$

Thus gladiator B captures gladiator A and wins.

Note: If all 3 corners of a gladiator's net end up at the same point, that gladiator is said to have captured himself and so lost.

See if you can devise a winning strategy for either gladiator.

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15's AND JAM

In the last issue of Parabola two games were given. You were told that the games were related to noughts and crosses.

The first game "15's" is quite easily seen to be equivalent to noughts and crosses if you managed to think of the 15's magic square.

2	9	4
7	5	3
6	1	8

The selecting of a digit is precisely the same as selecting an equivalent box at noughts and crosses. There are only eight ways that 3 distinct digits will sum to 15, each is a line in the magic square and thus equivalent to a win at noughts and crosses.

It was quite a deal harder to see the relationship between Jam and noughts and crosses. If the nine highways were numbered from 1 to 9 as in Figure 2, we notice that "Jam" is equivalent to "15's" as the sum of the highway numbers is 15 at each town. This in turn shows the equivalence of Jam to noughts and crosses.

There is also another way in which the relationship between Jam and noughts and crosses can be demonstrated. This is through the concept of dual diagrams.

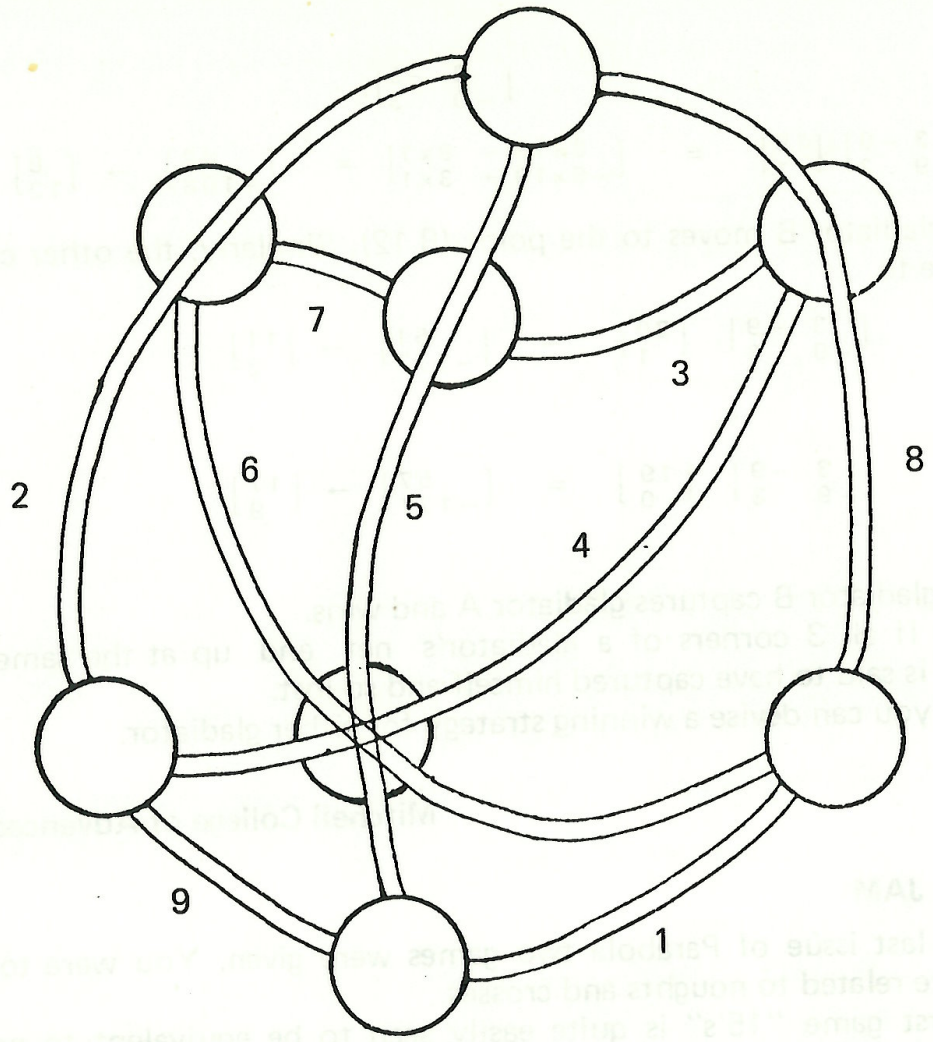
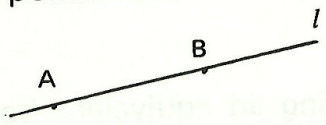


Figure 2

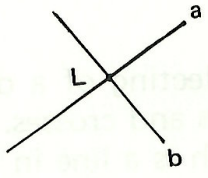
People have noticed that often the words "point" and "line" can be interchanged in plane geometry.

For instance:

Two points determine a line.

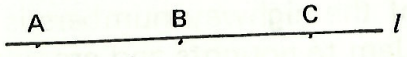


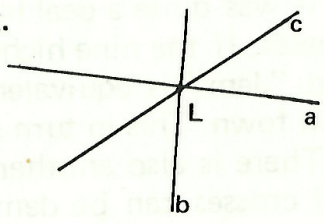
Two lines determine a point.



Sometimes other words have to interchange as well.

Three points lying on a line are said to be collinear.
Three lines "lying on" a point are said to be concurrent.





These above diagrams and statements are duals.

The dual of the noughts and crosses diagram is shown below. We see that this is in fact the Jam diagram.

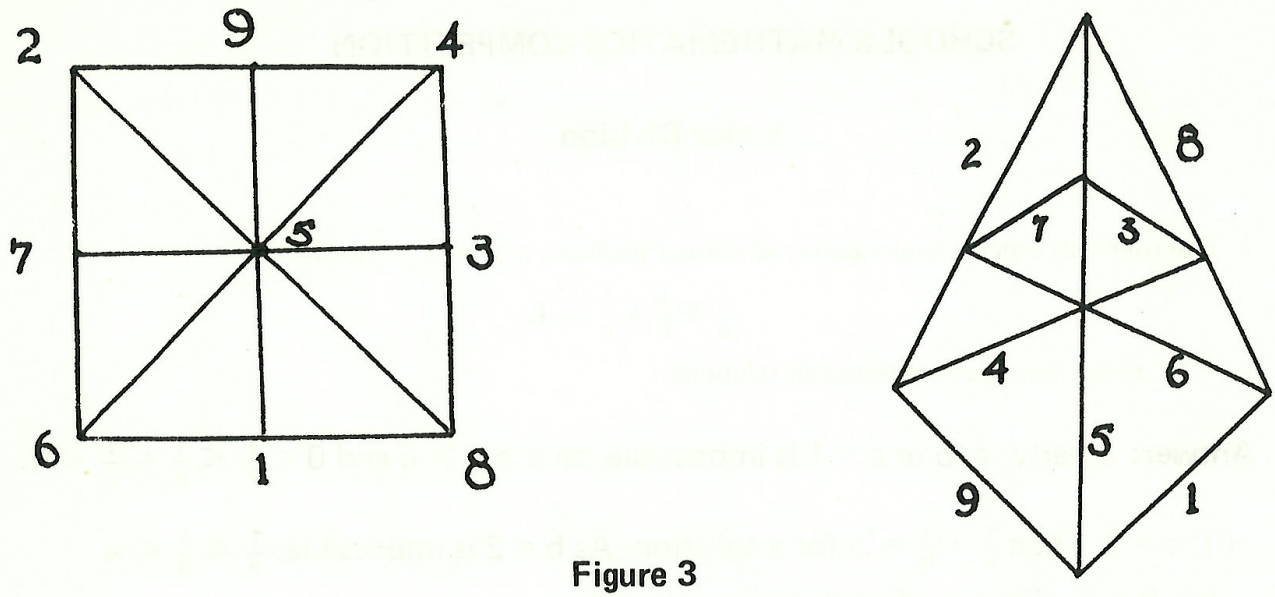


Figure 3

The dual of selecting a point in noughts and crosses is selecting a line in JAM.

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Mathematical Pen-friends

If you are interested in corresponding with some-one else who is interested in Mathematics, write to the Editor. When enough names and addresses have been received, a list will be sent to all who have written so that you may start writing to one another.

Reading Lists in Mathematics

Any readers who would like a reading list on some topics in Mathematics may be sent one on request. All we ask in return is that you read some of the books mentioned and write an article for Parabola.

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Solutions to coded messages in "Codes" (page 2)

JAMES BOND IS DEAD.
HE ONLY LIVED TWICE.
SEND MORE MONEY NOW.
HELD AT BEACH HOUSE.
THE END.