PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by August 1, 1974, will be published in the next issue Vol. 10 No. 3, 1974. Send all solutions to the Problem Editor (address on inside front cover).

The problems are classified as Junior, Intermediate or Open. Only students in the first two years of secondary education during 1974 are eligible to submit answers to the Junior problems, and only those in the first four years during 1974 may submit answers to the Intermediate problems. Anyone may submit solutions of problems in the Open section.

Junior

J241 By inserting brackets in

$$1 \div 2 \div 3 \div 4 \div 5 \div 6 \div 7 \div 8 \div 9,$$

the value of the expression can be made to equal 7/10. How?
Also find the largest value and the smallest value that can be obtained by insertion of brackets.

J242 Prove that the sum of two consecutive positive integers and the sum of their squares are relatively prime (i.e. have no common factor except 1).

Intermediate

I243 Prove that if $a(y+z) = x$, $b(z+x) = y$, $c(x+y) = z$ and at least one of the numbers x,y,z is not equal to zero, then

$$ab + bc + ca + 2abc - 1 = 0.$$

I244 A, B, C, D are points in a plane, no three of them being collinear, with AB = CD and AD = BC. Prove that, if the segments AB and CD intersect, then the segments AC and BD are parallel.

I245 Let p and q be successive odd prime numbers (i.e., if $p < k < q$, then k is composite). Show that $p+q$ has at least three (not necessarily distinct) prime factors. Further if $p \neq 3$ and $q = p + 2$ (i.e., p and q are twin primes) show that 6 is a factor of $p + q$.

Open

O246 Let p be an odd prime, let $p_1 = p + 2$, $p_2 = p_1 + 4$, . . . , $p_n = p_{n-1} + 2n$, . . . , the sequence continuing until a composite number is reached, e.g. $p = 5$, $p_1 = 7$, $p_2 = 11$, $p_3 = 17$, $p_4 = 25$. Is the sequence so generated always finite? If so, what is its maximum length, in terms of p?

O247 Of three cards, one is green on both faces, one white on both faces, whilst the third is green on one side and white on the other. They are placed in a hat, one is withdrawn and placed on a table. If the visible face is green, what is the probability that the other face is also green?

O248 Let P be any property defined on the set of positive integers $N = \{1,2,3,4, \ldots,\}$. (For example, P could be the property ". . . is even", or ". . . is odd", or ". . . is a prime number").
Let p_k denote the k'th positive integer (in order of magnitude) which has property P, and let $\pi(k)$ denote the number of natural numbers less than k having property P. (For example, if P is the property ". . . is even",

$$p_4 = 8 \text{ and } \pi(11) = \pi(12) = 5.)$$

Show that any positive integer r may be expressed either in the form $\pi(n) + n$ or else in the form $p_n + n$ for some positive integer n.

O249 Let $P(x) \equiv a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ be a polynomial with coefficients a_i which are all integers. If $P(x) = 0$ for 4 different integer values of x, show that the value of P(x) is never a prime number for any integer value of x.
Construct a polynomial with integer coefficients which vanishes for 3 distinct integer values of x and which has the value 11 at a fourth integer value.

O250 P is a point inside an acute angled triangle ABC. Where must P be so that the sum of the three distances PA, PB, and PC is least?