3 Dimensional Noughts and Crosses

The game of three dimensional noughts and crosses is played on four layers of 4 x 4 grids, as shown in Figure A. The play is the same as that of noughts and crosses except that four in a line is required for a win. This line can be either vertical, horizontal or diagonal, just as long as the four pieces are collinear.

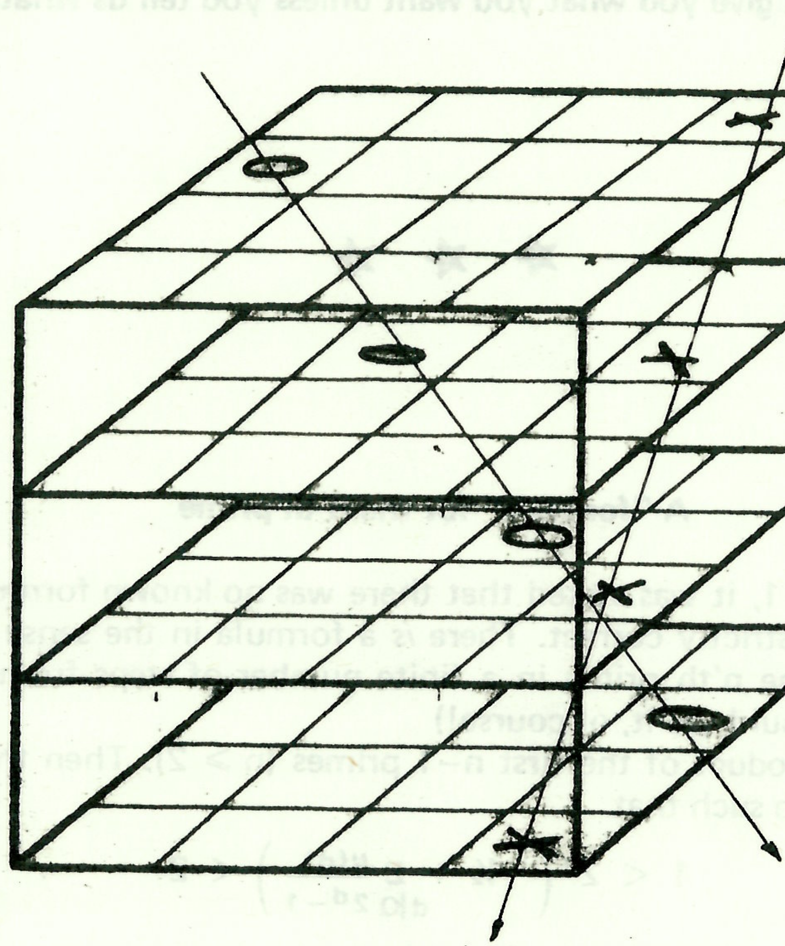

Figure A

You might care to build yourself a three dimensional noughts and crosses playing board. This can look very attractive if it is well constructed from perspex or some other transparent material. However, until you get around to doing this you can still play the game by ruling up the board as shown in Figure B.

You should be very careful about possible winning lines if you play this way, because some of them are very tricky to see.

Here are some questions you might like to try some rainy day:

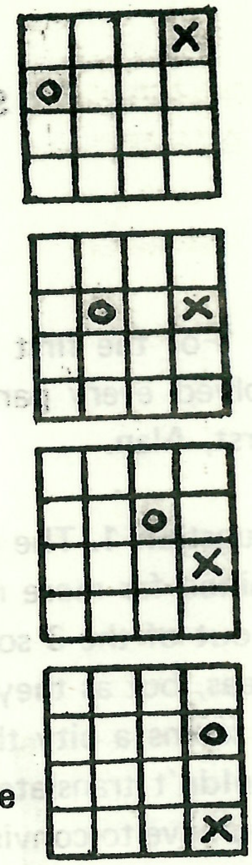

Figure B

1. Why is 3D noughts and crosses played on four layers of a 4 x 4 grid instead of the natural extension of 3 layers of the 3 x 3 grid?
2. There are eight possible winning lines in noughts and crosses. How many are there in 3D noughts and crosses?
3. There are only three significantly different first plays in noughts and crosses. How many are there in 3D noughts and crosses?

K.J.W.

Gladiators

While playing this game you may have noticed some interesting arrangements in the matrices used to determine the positions of the gladiators and their nets. The matrix $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ has an important value linked with it. This value is ad–bc and is called the determinant. You will have found that using some matrices that you will have lost because the net (i.e. triangle) had become a straight line or a point. If you test the determinant of the matrix in these cases you will find that it is zero and is to be avoided in future games. A useful matrix is $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ as this matrix leaves the net unchanged and can be used to place the net exactly as it was before the throw. If you wish to reflect about the main diagonal (i.e. the interval (0,0) to (20,20)) then you should use the matrix $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. This has the effect of interchanging the x and y coordinates of a point. It should be noted that the larger the value of the determinant, the larger the area of the "net". (This only applies if the net does not go over the edge of the area!)

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Note: One reader (David Powers of Fort Street Boys' High) sent a correct explanation for the relationship between the Mathematical Games in Vol. 10 No. 1. Congratulations, David.