SCHOOL MATHEMATICS COMPETITION 1974, EXAMINER'S COMMENTS

JUNIOR DIVISION

For the first time ever, in this division, the first prize winner, Alan Fekete, solved every part of every question correctly. Congratulations on your double first, Alan.

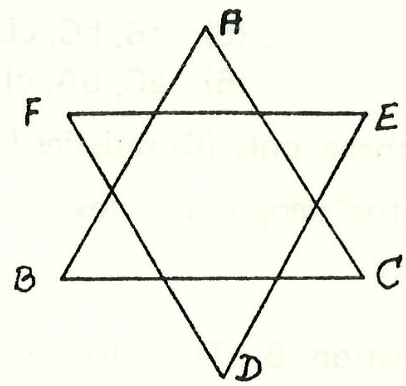
Question 1. The essential part of this question was the proof asked for, and this gained far more marks than the first part. Surprisingly, many candidates got only 2 out of the 3 solutions. In the second part I suspected that some had the right ideas, but as they didn't put them down clearly, they couldn't earn many marks. It seems a pity that they may have deprived themselves of success because they couldn't translate their ideas into words or mathematical symbols. Remember, you have to convince the examiner with your proof! Still, most students got a few marks out of this question.

Question 2. Competitors who proved that $2p + 1 = x^3$ worked for $x = 3$, $p = 13$ and didn't work for various other values of p or x which they substituted, gained no marks. The task was to prove that the equation worked only for $p = 13$ and not for any of the rest of the infinite set of prime numbers. Those who got as far as $p = \frac{1}{2}(x^3-1) = \frac{1}{2}(x-1)(x^2 + x + 1)$ or put $x = 2n + 1$ and $p = n(4n^2 + 6n + 3)$ got quite a few marks for an intelligent approach – and some sympathy (but no further marks!) from me for their bad luck in missing out on the last vital step, in the stress of the examination.

Question 3. Not many got either part right. Of those who did, most got the first part right by doing two or more pages of detailed working and then adding the individual results. Less than half a dozen competitors solved the second part or got near a solution of it. However I considered this question the most difficult of the six to get completely right and was not therefore surprised by the results.

Two competitors, from different examination centres, got the right answer to the first part but gave no explanation and showed no working. I gave them half marks or so; competitors must realise that any mathematics examiner in any mathematics exam is as interested in the method used as he is in the final answer Any working done is an essential part of the question, even rough working can help the examiner to understand the student's approach to a problem. I was able to give quite a few extra marks in various questions because of this fact. So, in future, hand in *everything* you do!

Question 4. I was agreeably surprised by the number who got this one correct. It was, of course, very much an "all or nothing" affair. I don't remember how I got on to the solution but I suppose one method I used was to try to fit two equilateral triangles on a diagram so that their corresponding vertices were one unit apart. Some students submitted the figure below, where AD, BE, CF are axes of symmetry of each of the two triangles. A valiant attempt, but unfortunately it won't work.



You can use trigonometry or geometry to show that if AB, BC, CA and AD, BE, CF are each one unit long then DE, EF, FD won't be one unit in length.

Question 5. This was the most popular question and more got this right than any other question. However, I did expect either an explanation of how you got the result or else a check, written down in the exam book, that the names you got did satisfy conditions (i), (ii), (iii).

Some students misinterpreted part (iii), judging by their answers.

An alternative proof of this question follows:—

Suppose the people are aX, bY, cZ, dT. Then X, Y, Z, T are all different.

(i) tells us that $X \neq A$, $Y \neq B$, $Z \neq C$, $T \neq D$.
(ii) tells us that $Z \neq A$.
So Z = B or D.
If Z = B, then X = C or D, Y = A, C or D, T = A or C.
so X = C, T = A, Y = D
or X = D, T = A, Y = C
or X = D, T = C, Y = A.
If Z = D, then X = B or C, Y = A or C, T = A, B or C.
so X = B, Y = A, T = C
or X = B, Y = C, T = A
or X = C, Y = A, T = B.

So there are six possible sets of names:

(1) aC, bD, cB, dA
(2) aD, bC, cB, dA
(3) aD, bA, cB, dC
(4) aB, bA, cD, dC
(5) aB, bC, cD, dA
(6) aC, bA, cD, dB

Of these, only (6) satisfies (iii).

So the people are alexander Charles, barry Alexander, charles David, and david Barry.

Question 6. This tied with Question 3 for difficulty in the eyes of most candidates, I think. The second part was a lot easier and shorter than the first part, however, and a correct diagram was a quite sufficient answer to it. Incidentally the solution to Part 2, given in the previous issue of Parabola, is by no means the only one.

With regard to the first part, many competitors mistakenly thought that all the houses had to lie on one path, while others proved that the minimum length was one kilometre – a true but irrelevant fact.

General Comments. As there is now a separate School Mathematics Competition in the A.C.T. we had very few competitors from Canberra this year. It is very pleasing to know that there are enough school students who are budding mathematicians there to sustain a separate Competition with its own separate questions and set of examiners. We have had some noteworthy prizewinners from Canberra in the past. Nevertheless, the general high standard has been maintained by candidates in 1974, despite the absence of Canberra students. Well done!

SENIOR DIVISION

Question 1. A simpler proof of the first part than the one in Parabola was supplied by certain entrants. Either $u \leqslant v$ or $v \leqslant w$ or $w \leqslant u$. In the first case,

$$u(1-v) \leqslant v(1-v) \leqslant \tfrac{1}{4}$$

and the others are similar.

Question 2. **(iii)** Another polynomial of degree 5 satisfying the requirement is x^5-x. In general, if p is any prime number and x is any integer then x^p-x is divisible by p. (This is known as Fermat's Theorem.)