PROBLEM SECTION

Students are invited to submit solutions of one or more of these problems. Answers should bear the author's name, class and school. Model solutions and the names of those who submit solutions by January 31, 1975, will be published in the next issue Vol. 11 No. 1, 1975. Send all solutions to the Editor (address on inside front cover).

The problems are classified as Junior, Intermediate or Open. The Junior problems are aimed at those students in the first two years of high school, the Intermediate problems are aimed at students in the first four years of high school and the Open problems are for all subscribers. Some problems are harder than others, so you are advised to submit solutions even if you can only solve one or two problems.

Junior

J251 Farmer Jones grew a square number of cabbages last year. This year he grew 41 more cabbages than last year and still grew a square number of cabbages. How many did he grow this year?

J252 I met triplets, A, B and C whose names were John, Peter, and Mick. When I asked who was who, A answered, "I'm not Peter."

B said, "I'm Peter."

C said, "I'm not John."

Then they told me that only one of them was telling the truth. Who was who?

Intermediate

I253 Is it possible to write the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 on a circle in such an order that the sum of two neighbouring numbers is never divisible by 3, 5, or 7?

I254 Solve the following simultaneous equations for x and y

$$xy(x-y) = ab(a-b)$$
$$x^3-y^3 = a^3-b^3.$$

I255 **(i)** Find a 6-digit number which is multiplied by the factor 6 if the final 3 digits are removed and placed (without changing their order) at the beginning.

(ii) Show that there is no 8 digit number which is increased by the factor 6 if the final 4 digits are transferred to the beginning.

Open

O256 Simplify the following expression

$$\left(1+\tfrac{1}{3}\right)\left(1+\tfrac{1}{9}\right)\left(1+\tfrac{1}{81}\right)\left(1+\tfrac{1}{3^8}\right)\ \ldots\ \left(1+\tfrac{1}{3^{2^n}}\right).$$

O257 The equation $x^5 + y^2 = z^4$ has $x = 2$, $y = 7$, $z = 3$ as a solution in integers. Are there any other solutions in integers?

O258 Show that if p is an odd prime, it divides the difference

$$[(\sqrt{5}+2)^p] - 2^{p+1}.$$

Note that $[x]$ means the integer n such that $n \leqslant x < n+1$.

O259 *(Submitted by M. Durie)* You have a beam balance, but no weights, and a collection of 12 similar coins, one of which however is counterfeit. It is a different weight from the good coins, but you do not know whether it is heavier or lighter. Locate the bad coin and its relative weight in three weighings.

O260 If n and k are any positive integers, show that

$$\frac{1}{n} - {}^kC_1\frac{1}{n+1} + {}^kC_2\frac{1}{n+2} + \ldots + (-1)^j\,{}^kC_j\frac{1}{n+j} + \ldots + (-1)^k\frac{1}{n+k}$$

is equal to $\dfrac{1}{l.c.m.\ \{D_0, D_1, \ldots, D_k\}}$ where D_j is the denominator when $\dfrac{{}^kC_j}{n+j}$ is put in lowest terms, $(j = 0, 1, 2, \ldots, k)$. $l.c.m.\ \{D_0, D_1, \ldots, D_k\}$ means the smallest number which $D_0, D_1, \ldots, D_k$ will all divide.

Solutions to Problems 241–250 in Vol. 10 No. 2.

Junior

J241 By inserting brackets in

$$1 \div 2 \div 3 \div 4 \div 5 \div 6 \div 7 \div 8 \div 9,$$

the value of the expression can be made to equal 7/10. How?
Also find the largest value and the smallest value that can be obtained by insertion of brackets.