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This column will be devoted to comment and discussion on some of the questions from recent Higher School Certificate examination papers. This time we look at some topics from the 1979 3-unit paper. In the next issue, we will turn to some 4-unit problems.

Problem 80.1. We are asked to find all real x for which $|x+1| > |x-1|$. This can be done by enumerating the four possibilities and then laboriously wading through the logic. (For example, if $x > -1$ and $x > 1$, then

$$|x+1| = x+1 > x-1 = |x-1|,$$

so the inequality holds for $x > 1$.) However, there is a much neater method. Let

$$X = |x+1| + |x-1|.$$

The two terms are greater than or equal to zero and, in fact, at least one of them is greater than zero for each x, so X is strictly positive for all real x. Now

$$X\{|x+1| - |x-1|\} = (x+1)^2 - (x-1)^2 = 4x,$$

that is

$$|x+1| - |x-1| = 4x/X.$$

Since $X > 0$, we deduce that $|x+1| - |x-1| > 0$ if and only if $x > 0$. Yet another attack involves a geometrical interpretation of the given inequality. Can you find it? Which do you think is the best method? To test your skill, find all real x for which $|x+2| > |x-1|$. Can you find all real x for which $|x+2| > 8|x-1|$?

Problem 80.2. A cargo boat travelling a fixed distance d km at constant speed v km per hour has an operating cost of $9000+10v^2$ dollars per hour. The problem is to determine v so that the operating cost for the journey is minimised.

It is important, as always, to read the question carefully and critically. There are several points to bear in mind. First, let $C = 9000+10v^2$ be the operating cost per hour. We are *not* asked to minimise C. Indeed, the minimum of C occurs when $v = 0$ ($dC/dv = 20v = 0$ when $v = 0$), an answer which is clearly absurd. A previous part of the question requires minimisation of the function $x+900/x$. This is a clue for the second part — do not be put off because x and v are different letters. Finally, remember that v is constant only for each particular trip, but can be varied from trip to trip, that is v is really the independent variable.

Now for the solution. The number of hours for the trip is d/v, so the total cost D, say, is $D = 10d(v+900/v)$. We have

$$dD/dv = 10d(1-900/v^2) = 0 \quad \text{when } v = \pm 30.$$

Since dD/dv is negative for $v < 30$ and positive for $v > 30$, the minimum value of D occurs when $v = 30$ km per hour. (Note that only positive values of D are relevant.)