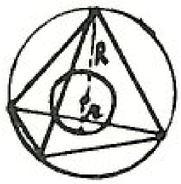
PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.



Q.793 The vertices of a regular tetrahedron lie on a sphere of radius R, and its faces are tangential to a sphere of radius, r. Calculate R/r.

Q.794 A and B are opposite vertices of a cube of side length 1 unit.

(i) Prove that the mid points of the six edges not containing either A or B all lie on a plane.

(ii) The cube is cut into two pieces along this plane. Find the radius of a sphere which touches that plane and three faces of the cube. (i.e. of the largest sphere which fits inside either of the two pieces.)

Q.795 If a, b, c are positive numbers such that $a^2 + b^2 = c^2$ prove that

$$\log_{c+b} a + \log_{c-b} a = 2 \log_{c+b} a \log_{c-b} a$$

Q.796 Find positive numbers x, y such that

$$\begin{cases} x^{x+y} &= y^{x-y} \\ x^2 y &= 1 \end{cases}$$

Q.797 Let $a_1, a_2, \cdots, a_n$ be any list of non-zero numbers such that for every $k \geq 3$

$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \cdots + \frac{1}{a_{k-1}a_k} = \frac{k-1}{a_1a_k}.$$

Prove that the list is an arithmetic progression.

Q.798 Prove that for every positive integer m,

$$\frac{1}{m+1} + \frac{1}{m+2} + \frac{1}{m+3} + \cdots + \frac{1}{3m+1} > 1$$

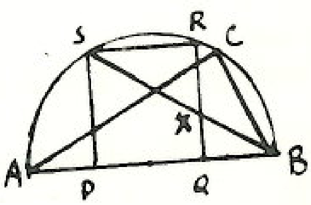
Q.799 Two identical containers, A and B, each of capacity L litres, together contain a total of L litres of alcohol. First A is filled to the top with water, and the contents stirred thoroughly. Then liquid is poured from A into B until it is full. Finally $(2/5)L$ litres of the new mixture in B is transferred to A without spillage. If A now contains $\frac{L}{15}$ litres more alcohol than B, find how many litres of alcohol were in each container originally.

Q.800 Two bodies move in opposite directions, one at constant speed v m/sec. The speed of the other increases at a constant rate a m/sec/sec. At time $t = 0$, the bodies are of the same point A and the second one is momentarily at rest. In how many seconds does their first meeting occur, if their second meeting is again at the point A?

Q.801 Show that it is impossible to find three whole numbers u, v, w such that

$$u^2 + v^2 = 3w^2$$

Q.802 How many different triangles (i.e. no two congruent) have perimeter 150cms, and every side a whole number of centimetres in length?



Q.803 In the figure $ABCRSA$ is a semicircle and $PQRS$ is a square.
Area $\triangle ABC$ = Area $PQRS$.
Prove that X, the point of intersection of BS and QR, is the incentre of $\triangle ABC$.

Q.804 $a_0, a_1, a_2, a_3, \cdots, a_n, \cdots$ is an increasing sequence of real numbers ($a_n < a_{n+1}$ for all n) defined by $a_{n+1} = 2^n - 3a_n$, $n = 0, 1, 2 \cdots$.
Find all possible values of a_0.

Solvers of Problems 782 - 792

T. Duke (Shore School):- Q.783

P.S. Maylott (Sydney Grammar):- Q.782, Q.783, Q.784, Q.786, Q.788, Q.789

R. Rodriguez (A.E.M. Science H.S. Philippines):- Q.782

S.G. Toribio (A.E.M. Science H.S. Philippines):- Q.785, Q.792.