SOLUTIONS TO THE COMPETITION PROBLEMS

JUNIOR

1. (a) The new teacher's age is an odd number which leaves the remainder 1 when divided by 3, and the remainder 9 when divided by 11.
How old is the teacher?

(b) By coincidence, her street number also has these properties. Furthermore, the street number has three digits, and when it is divided by 7 the remainder is 6. What is the street number?

ANSWER. Since her age, A, is odd, set $A = 2n + 1$ where n is a whole number. When n is divided by 3 let the quotient be q and the remainder be r. Then $r = 0$, or 1, or 2.

$$n = 3q + r \quad \text{and} \quad A = 6q + 2r + 1.$$

If $r = 1$, $A = 3(2q + 1)$, and if $r = 2$, $A = 3(2q + 1) + 2$. In neither of these cases does A leave the remainder 1 when divided by 3. Hence we must have $r = 0$, $A = 6q + 1$.

Now let $q = 11Q + R$ where $0 \leq R < 11$. Then $A = 66Q + (6R + 1)$. Trying $R = 0, 1, \cdots, 10$ we find that $6R + 1$ leaves the remainder 9 on division by 11 only for $R = 5$. Hence $A = 66Q + 31$ for some integer Q. For $Q = 0$, $A = 31$ and we conclude that this is the new teacher's age, since the next possibility, $Q = 1$, $A = 97$, is well past retirement age.

ii) Her street number, S, is also given by

$$S = 66Q + 31.$$

Let $Q = 7q' + r'$ where $r' = 0$, 1, 2, 3, 4, 5 or 6. $S = 7 \times 66q' + 66r' + 31$.
Now $66r' + 31 = 7 \times (9r' + 4) + 3r' + 3$ which leaves the remainder 6 on division by 7 only when $r' = 1$. Therefore $S = 462q' + 97$. For $q' = 0, 1$, or 2, this formula gives $S = 97, 559$ and 1021 respectively. Therefore the only 3-digit possibility for S is 559.

2. Four friends Alan, Anne, Belinda and Brian whose surnames in some order are Carter, Clark, Davidson, and Dwyer have just received their examination marks in Mathematics and English. The four English marks are 50,60,70, and 80 and the mathematics marks are 60,70,80,90. When the marks were added, the highest total was obtained by a girl. In each subject the highest mark was obtained by a boy, and the lowest mark by someone with the initial D. In English Belinda beat Carter, and Brian beat Dwyer. Find the complete names and marks of all four.

ANSWER. The boys cannot have obtained a total of less than 140 marks (best in one subject, worst in the other), so the girl who obtained the highest total must have obtained 150 by getting the second best mark in each subject. Since the boys scored less than this, they must indeed have obtained the lowest marks in both subjects, so their names are Davidson and Dwyer. Hence Belinda, not being Carter, must be Belinda Clark who obtained 70 marks for English and 80 marks for Mathematics. Brian Davidson obtained 80 for English and 60 for Mathematics, and Alan Dwyer obtained 50 for English and 90 for Mathematics. Finally, Anne Carter obtained 60 for English and 70 for Mathematics.

3. A tiler has a number of square floor tiles, all the same size, of which exactly one third are black. He can use them to tile either a rectangular room or a square room (in either case using all the tiles without cutting) but decides on the former because there is exactly the right number of black tiles to go round the edge of the room. How many tiles does he have?

ANSWER. Let the dimensions of the rectangular room be a units long by b units wide (where one unit is the side length of a tile). Since there is a course of black tiles round the edge, the other tiles occupy a rectangle of dimensions $(a-2)$ by $(b-2)$.

$$\therefore (a-2)(b-2) = \frac{2}{3}ab$$

$$ab - 6a - 6b + 12 = 0$$

$$a = \frac{6b-12}{b-6} = \frac{6b-36}{b-6} + \frac{24}{b-6}$$

$$= 6 + \frac{24}{b-6}$$

Remembering that a and b are whole numbers with $a > b$, and hence that $b-6$ must be an exact divisor of 24, the possible values of a and b are:-

$$b-6=4; \quad a=12,\ b=10$$

$$b-6=3; \quad a=14,\ b=9$$

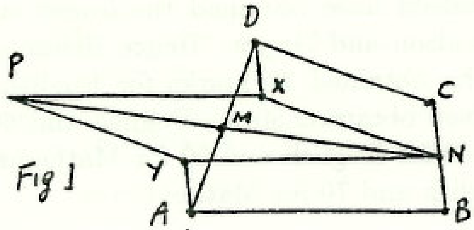
$$b-6=2; \quad a=18,\ b=8$$

$$b-6=1; \quad a=30,\ b=7.$$

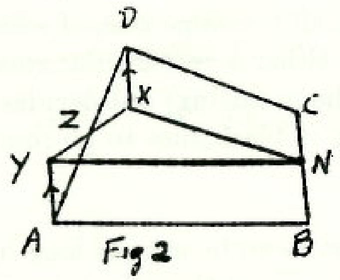
The total number of tiles, $a \times b$, is 120, 126, 144, or 210 in these four cases. Since a square room could be tiled without cutting, the number of tiles must be a perfect square. Hence the tiler must have 144 tiles altogether.

4. $ABCD$ is a convex quadrilateral, and M, N are the midpoints of the sides AD and BC respectively. If it is given that $MN = \frac{1}{2}(AB + CD)$ prove that the quadrilateral is a trapezium.

ANSWER. Given $BN = NC; AM = MD$;
$2MN = AB + CD$.
Prove $AB \| CD$.



Suppose AB is not parallel to CD. Construct X, Y such that $NCDX$ and $ABNY$ are parallellograms. Join XY. Let XY cut AD at Z. (Fig. 2)
Since $AY \| BC \| DX$.



$$Y\hat{A}Z = X\hat{D}Z \text{ (alternate angles)}$$

Also $Y\hat{Z}A = X\hat{Z}D$ vertically opposite

and $YA = XD$ (both $= \frac{1}{2}BC$.)

$\therefore \Delta YZA \equiv \Delta XZD$

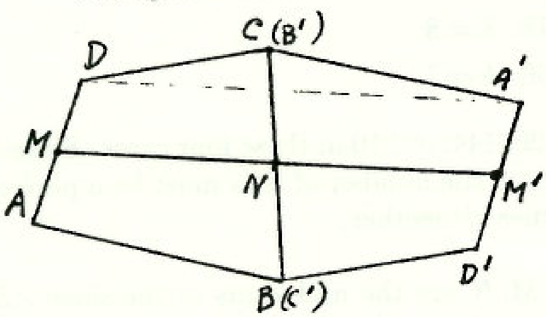
Hence $ZA = ZD$.

Thus Z coincides with M, the mid-point of AD. Since $YZ = XZ, M$ also bisects XY. Now produce NM to P so that $MP = NM$. Then $NXPY$ is a parallelogram, since the diagonals XY and NP bisect each other.

$$\therefore \text{From triangle } NXP, \; NP < NX + XP$$
$$2NM < NX + NY$$
$$2NM < CD + AB.$$

(Since $CD = NX$ and $AB = NY$, opposite sides of parallellograms). This contradicts the data, so our supposition that AB is not parallel to CD must be false. Therefore $AB \| CD$ and $ABCD$ is a trapezium.

ALTERNATIVE ANSWER. Rotate the quadrilateral about the point N through 180°, into position $A'B'C'D'$ as shown.



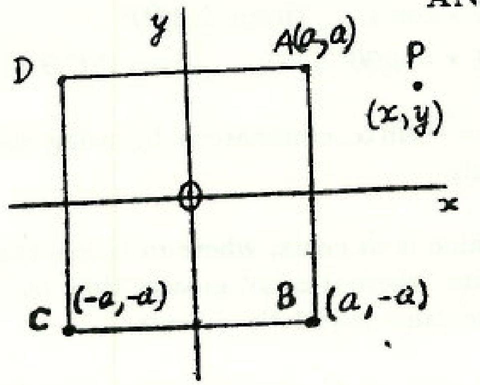
Since N is the mid point of BC, B' coincides with C, and C' with B. Since $M\hat{N}M' = 180°, M\ M'$ is straight. Since the rotation is through 180°, every line segment moves to a parallel line segment. Thus $AD \| D'A'$. Since MD and $M'A'$ are equal and parallel, $MM'A'D$ is a parallellogram.

$$\therefore A'D = M'M = 2MN = AB + CD = A'B' + CD$$
$$A'C + CD.$$

Thus the point C must lie on the straight line $A'D$; whence $A'B' \| AB \| CD$, as required.

5. (a) A point is located outside a square. The distance from the point to the nearest corner of the square is 5 units; to the next nearest, 11 units, and to the farthest, 17 units. Find the area of the square.

(b) As above, but the point is inside the square.



ANSWER. Take axes parallel to the sides of the square, with the origin at its centre point. We may take $P(x,y)$ with $0 < y \le x$ as the point in question. Let the side of the square be of length 2a, so that the co-ordinates of its vertices are $(\pm a, \pm a)$.

$$\text{Then } PA^2 = (x-a)^2 + (y-a)^2 = 25 \quad (1)$$
$$PB^2 = (x-a)^2 + (y+a)^2 = 121 \quad (2)$$
$$PC^2 = (x+a)^2 + (y+a)^2 = 289 \quad (3)$$

From (1) and (2)

$$(y+a)^2 - (y-a)^2 = 121 - 25$$
$$4ay = 96, \; y = \frac{24}{a}. \qquad (4)$$

From (2) and (3)

$$(x+a)^2 - (x-a)^2 = 289 - 121$$
$$4ax = 168, \; x = \frac{42}{a}. \qquad (5)$$

Substituting for x and y in (1), using (4) and (5) gives

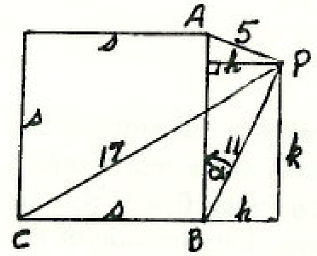
$$(\frac{42}{a} - a)^2 + (\frac{24}{a} - a)^2 = 25.$$

After simplification

$$2a^4 - 57a^2 + 2340 = 0$$
$$(2a^2 - 117)(a^2 - 20) = 0$$

So the area of the square, $4a^2$, is either 234, or 80.
If $a^2 = 20$, $ay = 24 > a^2 \Rightarrow y > a$ and similarly $x > a$. The point (x,y) is outside the square. If $a^2 = \frac{117}{2}$, $ax = 42 < a^2 \Rightarrow 0 < y < x < a$. The point (x,y) is inside the square. Thus the answers are (a) 80 (b) 234.

ALTERNATIVE ANSWERS (1). Not using analytic geometry, but still using Pythagonas theorem.



$h^2 + (s-k)^2 = 5^2;\ h^2 + k^2 = 11^2; (h+s)^2 + k^2 = 17^2$ etc.

(2). Using trigonometry

$5^2 = s^2 + 11^2 - 2 \times 11 \times \cos\alpha$ (from $\triangle ABP$

$17^2 = 2^2 + 11^2 - 2 \times 11 \times \cos(90° + \alpha)$ (from $\triangle CBP$)

Since $\cos(90° + \alpha) = -\sin\alpha$, eliminate α by using $\sin^2\alpha + \cos^2\alpha = 1$ to obtain an equation in s only.

6. I have k postage stamps whose total value is m cents, where m is less than $2k$. The postage required on a letter is n cents, where n is not greater than m. Prove that it is possible to select stamps whose value is exactly n cents.

ANSWER. If k is a very small number, say 1, or 2, or 3, one can easily note that the result is true by considering all possibilities. e.g. if $k = 3$, the possible stamp values are 1, 1, 1 ($m = 3$); or 1, 1, 2 ($m = 4$); or 1, 2, 2 ($m = 5$); or 1, 1, 3 ($m = 5$); and in each case any postage up to m cents is exactly payable. Our strategy is to show that for any k, the problem can be reduced to a similar problem with a smaller k. Eventually it reduces to a problem with k equal to 3 or less, and hence a solution always exists.

(*) Note first that in the set of stamps, no stamp has a value which exceeds by 2 (or more) the total value, V, of all K smaller stamps. For the average value of these $K+1$ stamps is not less than $\dfrac{V + (V+2)}{K+1} \geq \dfrac{2V+2}{V+1}$ (since $V \geq K$) ≥ 2. This contradicts the datum $m < 2k$ which insures that the average value of all the stamps (and hence certainly of the $K+1$ smallest stamps) is less than 2. Now consider the general situation. If n does not exceed the number of 1 cent stamps, it is obvious that n 1 cent stamps may be affixed. Otherwise affix to the letter the largest stamp whose value v, does not exceed n. Because of the note (*), we must have $v \geq 2$. (If $v = 1$, all K stamps of value $\leq n$ have value 1 cent, but $n \geq K+1$. The next smallest stamp, with value $> n$, has value $\geq K+2$ contradicting (*)).

Having affixed this stamp, there remain $k-1$ stamps with total value $m' = m - v \leq m - 2 < 2(k-1)$, and we have yet to pay postage of $n' = n - v \leq m'$. This has reduced the problem to a smaller problem, still satisfying the same conditions, from which it follows that the correct postage is always payable.

7. From the list, $1, 2, 3, \cdots, 100$ I want to choose three different numbers which add up to 100. (The order of selection is not important; the choice $\{20, 30, 50\}$ is the same as $\{30, 50, 20\}$ for example).

In how many different ways can the selection be made

(a) if the smallest number is 25?

(b) with no such restriction?

ANSWER. (a) The middle sized number must be one of 26, 27, $\cdots$, 37 (since it must exceed 25, but be less than $\frac{1}{2}(100-25)$).
Hence there are 12 ways of making the selection.

(b) Let N_n be the number of ways of making the selection with the smallest number chosen equal to n. Thus $N_{25} = 12$ by (a).
Arguing as in (a), if n is an odd number, the middle sized number must be one of $n+1,\ n+2,\ \cdots \frac{1}{2}(99-n)$. Therefore $N_n = \frac{1}{2}(99-n) - n = \frac{1}{2}(99-3n)$.
If n is even, the middle sized number is one of $n+1,\ n+2, \cdots, \frac{1}{2}(98-n)$. Therefore $N_n = 49 - \frac{3}{2}n$. The total number of ways the selection can be made is

$$\begin{aligned}(N_1+N_2)+(N_3+N_4)+\cdots+(N_{31}+N_{32})\\ &= (48+46)+(45+43)+\cdots+(3+1)\\ &= (2\times 48-2)+(2\times 45-2)+\cdots+(2\times 3-2)\\ &= 6\times(16+15+14+\cdots+1)-16\times 2\\ &= 784.\end{aligned}$$

SENIOR

1. 573 has the property that its square can be written down as two consecutive integers

$$573^2 = 328,329$$

Find all three digit numbers with this property.

ANSWER. If x is such a number, then

$$\begin{aligned}x^2-1 &= abc,\ abc\\ &= y\times 1001(\text{where } y = 100a+10b+c)\\ (x-1)(x+1) &= y\times 7\times 11\times 13.\end{aligned}$$

Since $x+1 < 1001$ at most two of the factors 7, 11, 13 divide $x+1$, the other(s) divide(s) $x-1$. Try

$$\left.\begin{aligned}x-1&=77k\\ x+1&=13\ell\end{aligned}\right\}(k<13)$$

$13\ell = 77k+2 = 13\times 6k - k + 2$ which is divisible by 13 if $k=2$; $x = 154+1 = 155$. However $x^2 = 155^2 = 24025 = 024025$ which answers the question only if we allow ourselves the privilege of leading zeros.

$$\text{Try} \quad x+1 = 77k$$
$$x - 1 = 13\ell$$

Then $77k-2 = 13\times 6k - k - 2$ which is divisible by 13 if $k = 11$, $x = 11\times 77 - 1 = 846$

$$846^2 = 715716.$$

Similarly $x - 1 = 91k$, $x + 1 = 11\ell, (k < 11)$

$$\Rightarrow 91k + 2 = 11 \times 8k + (3k + 2) = 11\ell$$
$$\Rightarrow k = 3 \Rightarrow x = 274.$$
$$274^2 = 075076.$$

$x + 1 = 91k$, $x - 1 = 11\ell \Rightarrow 11\ell = 91k - 2 =$

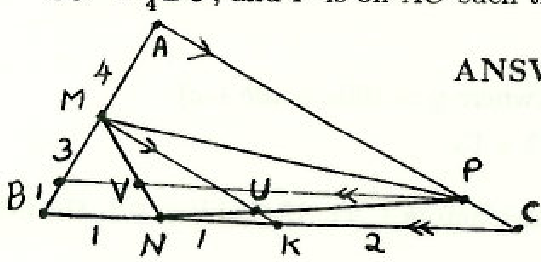
$$11 \times 8k + (3k - 2) \Rightarrow k = 8 \Rightarrow x = 8 \times 91 - 1 = 727$$
$$727^2 = 528,\ 529$$

Again $x - 1 = 143k$, $x + 1 = 7\ell$
$\Rightarrow 7\ell = 143k + 2 = 7 \times 20k + (3k + 2) \Rightarrow k = 4 \Rightarrow x = 4 \times 143 + 1 = 573$ or
$x + 1 = 143k$, $x - 1 = 7\ell \Rightarrow 7\ell = 143k - 2 = 7 \times 20k + (3k - 2)$
$\Rightarrow k = 3 \Rightarrow x = 3 \times 143 - 1 = 428.$

$$428^2 = 183,184.$$

Thus the four values of x are 816, 727, 573, or 428 (and the values $x = 155$, and 274 are also of interest, although not quite respectable as solutions of the problem).

2. Given 3 points M, N, P not in a straight line, show how to construct a triangle ABC such that M is the midpoint of side AB, N is a point on BC such that $BN = \frac{1}{4}BC$, and P is on AC such that $CP = \frac{1}{8}CA$.



ANSWER. Let ABC be the correct triangle, and consider a line through M parallel to AC, cutting BC at K. Then K is the midpoint of BC so $\frac{NK}{KC} = \frac{1}{2}$
Therefore if MK intersects NP at U we have $\frac{NU}{UP} = \frac{1}{2}$.

A similar calculation shows that a line through P parallel to BC cuts MN at V where $\frac{NV}{VM} = \frac{1}{3}$.
Hence, given MNP, construct U on NP such that $\frac{NU}{UP} = \frac{1}{2}$, and V on NM such that $\frac{NV}{VM} = \frac{1}{3}$. Through N construct a line parallel to PV cutting MU (produced) at K. Construct B on this line so that $BN = NK$, and C such that $KC = BK$. Construct lines BM and CP intersecting at A.

3. (a) A point is located outside a square. The distance from the point to the nearest corner of the square is 5 units; to the next nearest, 11 units, and to the farthest, 17 units. Find the area of the square.

(b) As above, but the point is inside the square.

ANSWER. Refer to the answer to Junior question 5.

4. A function is defined when $m, n = 0, 1, 2, \cdots$ by the following rules

(1.) $A(0, m) = m + 1$

(2.) $A(n + 1, 0) = A(n, 1)$

(3.) $A(n + 1, m + 1) = A(n, A(n + 1, m))$.

Prove that $A(1, m) = m+2$ and that $A(2, m) = 2m+3$ and find a formula for $A(3, m)$.

ANSWER. By 3;

$$\begin{aligned} A(1, m) &= A(0, A(1, m - 1)) \\ &= 1 + A(1, m - 1) \quad \text{by (1)}. \end{aligned}$$

Repeating this m times, we obtain

$$\begin{aligned} A(1, m) &= \underbrace{1 + 1 + 1 + \cdots + 1}_{m \text{ terms}} + A(1, 0) \\ &= m + A(0, 1) \quad \text{by (2)}. \end{aligned}$$

$$A(1, m) = m + 2 \quad \text{by (1)} \tag{4}$$

Again,

$$\begin{aligned} A(2, m) &= A(1, A(2, m - 1) \quad \text{by (3)} \\ &= 2 + A(2, m - 1) \quad \text{by (4)} \end{aligned}$$

Repeating this m times gives

$$\begin{aligned} A(2, m) &= \underbrace{2 + 2 + \cdots + 2}_{m \text{ terms}} + A(2, 0) \\ &= 2m + A(1, 1) \quad \text{by (2)} \end{aligned}$$

$$A(2, m) = 2m + (1 + 2) = 2m + 3 \tag{5}$$

Finally

$$\begin{aligned} A(3, m) &= A(2, A(3, m - 1)) \quad \text{by (3)}. \\ &= 3 + 2 \times A(3, m - 1) \quad \text{by (5)} \end{aligned}$$

Repeating yields

$$\begin{aligned}
A(3,m) &= 3 + 2(3 + 2A(3,m-2)) \\
&= 3 + 2\times 3 + 2^2(3 + 2A(3,m-3)) \\
\text{(after } m \text{ times)} &= 3 + 2\times 3 + 2^2\times 3 + 2^3\times 3 + \cdots + 2^{m-1}\times 3 + 2^m\times A(3,0) \\
&= 3(1 + 2 + 2^2 + \cdots + 2^{m-1}) + 2^m \times A(2,1) \\
&= 3(2^m - 1) + 2^m \times (2\times 1 + 3) \text{ by (5)} \\
&= 8\times 2^m - 3
\end{aligned}$$

5. We select from the set $\{1, 2, \cdots, 179\}$ three different numbers at random. What is the probability that they form the angles of a triangle (measured in degrees; i.e., they add up to 180)?

ANSWER. The probability is $\frac{N}{D}$ where N is the number of selections of three numbers from the set which add to 180, and D is the total number of ways of choosing 3 numbers from the set.

Thus $D = {}^{179}C_3 = \dfrac{179\times 178\times 177}{3\times 2\times 1} = 939929.$

To calculate N we may proceed in a manner similar to that used in the seventh question on the Junior paper, or varying slightly, as follows:- Let $a < b < c$ be the three numbers selected and let N_b denote the number of different selections with the middle sized number equal to b. Note that b can be any one of 2,3,4$\cdots$, 89. If $b \le 60$, a can be any one of $1, 2, \cdots, b-1$, so that $N_b = b - 1$.
If $b \ge 60, c$ can be any one of $b+1, b+2, \cdots, 179-b$ so that $N_b = 179 - 2b$.

$$\begin{aligned}
\therefore N &= N_2 + N_3 + N_{60} + N_{61} + \cdots + N_{89} \\
&= 1 + 2 + \cdots 58 + 59 + 57 + 55 + \cdots + 3 + 1 \\
&= \frac{59\times 60}{2} + \frac{38\times 29}{2} = 2611 \\
\therefore \text{ Probability } &= \frac{2611}{939929} = 0.278\%
\end{aligned}$$

Alternatively, in the above, N may be calculated cleverly by

$$N = \frac{{}^{179}C_2 - 3\times 89 + 2}{3!} = \frac{\frac{179\times 178}{2} - 3\times 89 + 2}{6} = 2611.$$

The first term on the numerator is the total number of solutions of $x+y+z = 180$ where x, y and z are positive integers. (Imagine 180 matchsticks in a row. There are 179 spaces between them. Every choice of two of these 179 gaps separates the matches into 3 groups and corresponds to a solution of $x + y + z = 180$). This count includes solutions with two numbers equal. There are 89 solutions with $x = y$, since x can be any of $1, 2, \cdots 89$. Similarly there are 89 solutions with $x = z$ and 89 with $y = z$. The middle term has subtracted these unwanted solutions. But the solution $x = y = z = 60$ (the equilateral triangle) has been subtracted three times, instead of just once. Hence the final term +2. Now the numerator counts all solutions of $x + y + z = 180$ with x, y, z all different; but each of the 3! different arrangements has been counted as a new solution.

6. In a party of m people, among any three there are at least two who know each other.

(a) If $m = 6$ prove that there are 3 people each of whom knows the other 2.

(b) If $m = 10$ prove that in the party there are 4 people each of whom knows the other 3.

(c) If $m = \frac{1}{2}n(n+1)$ prove that in the party there are n people mutually acquainted.

(d) If $m = 9$ prove that in the party there are 4 people mutually acquainted.

(e) Show that if $m = 8$ there need not be any 4 people mutually acquainted.

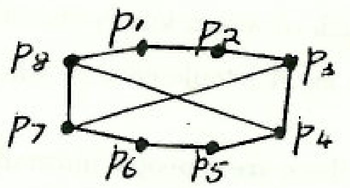
ANSWER (a) Let A be one of the people. If A knows three (or more) of the other five, then two of these three (X, Y say) know each other. Then $\{A, X, Y\}$ are mutually acquainted. Otherwise there are at least three of the five others $\{U, V, W\}$ say none of whom knows A. From the trio $\{A, U, V\}$ we see that U and V must be acquainted. Similarly for U, W and V, W. Hence $\{U, V, W\}$ are mutually acquainted.

(b) Replace each person by a labelled point and join two points with a red line if the corresponding people are acquainted, and with a green line otherwise. The data translates into the assertion that there is no green triangle in the resulting figure. Let A be any one of the ten points (i.e. people). The other 9 separate into X : those connected to A by a green line; and Y : those connected to A by a red line. If there are four or more points in X then every pair of them must be connected by a red line (to avoid a green triangle with A); hence any 4 people from X know each other. But if X contains at most 3 points, then Y must contain 6 (or more). By (a) we can find 3 points in Y which are joined by red lines, and with A we have again found 4 people who are mutually acquainted.

(c) (a) and (b) were the cases $n = 3$ and $n = 4$. We prove the general result by mathematical induction. Assume it is true that if $m = \frac{1}{2}(n-1)n$ there are $(n-1)$ people mutually acquainted. Consider any party of $\frac{1}{2}n(n+1)$ people (or points) and let A be one of them.

If the set X of points joined to A by green lines contains n (or more) points, any two of these, PQ, must be joined by a red line (otherwise APQ is a green triangle), so the group X satisfies the required conditions. But if X has at most $(n-1)$ points then Y contains $\frac{1}{2}n(n+1) - (n-1) - 1$ points (at least). Since this number is $\frac{1}{2}(n-1)n$, by our induction assumption there is a subset of $(n-1)$ points in Y all connected by red lines, and with A again we have a set of n people mutually acquainted.

(d) As in (b) if A is joined to 4 other points by green lines, we are finished since these 4 points are all joined by red lines. So suppose every point has at most 3 green lines leaving it. It is impossible that exactly 3 green lines end at every vertex, since that would mean the green lines have altogether $9 \times 3 = 27$ ends, which should (but can't) be twice the number of green lines in the diagram. Hence some point, B say, has at most 2 green lines leaving it. Then B has 6 (or more) red lines, and as before, by (a), one can find 3 of the people known to B who also know one another.



(e) The diagram has represented people by points, and two points have been joined by a line if the two people are <u>not</u> acquainted. It can be checked that no three points are all joined.

We seek a subset of 4 vertices not containing any side. Since all sides of the octagon are drawn, there are only two possibilities $\{p_1p_3p_5p_7\}$ or $\{p_2p_4p_6p_8\}$. But p_3p_7 and p_4p_8 are unacquainted pairs in these sets. Hence there are no 4 people mutually acquainted in this party of 8.

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"Once when lecturing to a class he [Lord Kelvin] used the word "mathematician," and then interrupting himself asked his class: "Do you know what a mathematician is?" Stepping to the blackboard he wrote upon it:-

$$\int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

Then putting his finger on what he had written, he turned to his class and said: "A mathematician is one to whom *that* is as obvious as that twice two makes four is to you. Liouville was a mathematician"

S.P. Thompson, **Life of Lord Kelvin.**