Parabola Volume 26, Issue 2 (1990)

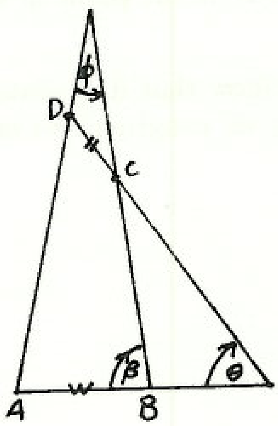
PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

Q.805 Solve for x and y:

$$\sqrt{x+y}+\sqrt{x-y}=5\sqrt{x^2-y^2}$$

$$\frac{2}{\sqrt{x+y}}-\frac{1}{\sqrt{x-y}}=1$$



Q.806 Show how to construct a quadrilateral $ABCD$, being given the lengths AB and CD, the angle $A\hat{B}C$, and the angles made by producing both pairs of opposite sides.

Q.807 A confirmed clock watcher drew lines on the face of a clock to mark the positions of the two hands. Some time later he noticed that the hands now trisected the angle between the marked lines. In how short a time could this have happened? How soon after 3.00 could one start such an experiment?

Q.808 Let $b_1, b_2, ..., b_n$ be any rearrangement of the positive numbers $a_1, a_2, ..., a_n$. Prove that $\frac{b_1}{a_1}+\frac{b_2}{a_2}+...+\frac{b_n}{a_n}\geq n$.

Q.809 How many different (i.e. non congruent) triangles can be constructed with side lengths in cms a, b, c given that a, b, c are all whole numbers, and $a<b<c\leq n$ where n is a given whole number.

Q.810 Solve for x the equation

$$\sin(18+\frac{3x}{2})^0=2\sin(54-\frac{x}{2})^0$$

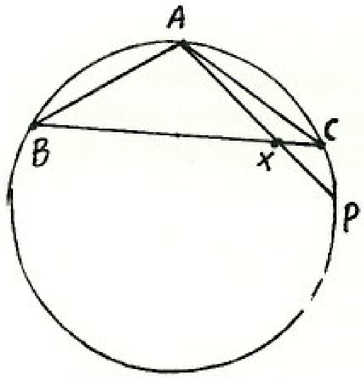
Q.8111 All the digits of a whole number x are different, and they are in increasing order. Let $S(x)$ denote the sum of all the whole numbers which can be obtained by rearranging the digits of x (including x itself). Thus, if $x=378$, $S(x)=378+387+738+783+837+873=3996$. Find all numbers x such that $S(x)=86658$.

Q.812 $P(x)$ is a polynomial with integer coefficients, and n, m are positive whole numbers.

Given that (i) $19>n>m$

(ii) $P(m)=0,\ P(n)=39,\ P(19)=187$

find m and n.



Q.814 The list of numbers $a_1, a_2, a_3, ..., a_k, ...$ is defined by $a_1 = 1$, and $a_k^2 + a_{k+1}^2 = 2(\frac{a_k + a_{k+1}}{2} + 1)^2$ for $k \geq 1$.

(i) Prove that all terms in the list are whole numbers.

(ii) Find a_{1990}.

Q.815 The isosceles triangle ABC is obtuse angled at A and P is a point on the circumcircle such that $PB + PC = 2AB$.

Prove that $2AX = BC$. (where X is the point of intersection of AP and BC.)

Q.816 Let $P_1, P_2, ..., P_n$ be any n distinct points in the plane. Show that it is always possible to dissect the plane into n regions $R_1, R_2, ..., R_n$ all congruent to one another, such that P_i lies inside R_i for $i = 1, 2, ..., n$.

* * * * * * * * * * *

"Mathematics has a triple end. It should furnish an instrument for the study of nature. Furthermore it has a philosophic end, and, I venture to say, an end aesthetic. It ought to incite the philosopher to search into the notions of number, space, and time; and, above all, adepts find in mathematics delights analogous to those that painting and music give. They admire the delicate harmony of number and of forms; they are amazed when a new discovery discloses for them an unlooked for perspective; and the joy they thus experience, has it not the aesthetic character although the senses take no part in it? Only the privileged few are called to enjoy it fully, it is true; but is it not the same with all the noblest arts? Hence I do not hesitate to say that mathematics deserves to be cultivated for its own sake, and that the theories not admitting of application to physics deserve to be studied as well as others."

Henri Poincaré.