PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

Q.852 If $a_1, a_2, \cdots a_n$ are positive real numbers and $a_1 + a_2 + \cdots + a_n = 1$ prove that

$$\sum_{k=1}^{n} \left(a_k + \frac{1}{a_k} \right)^2 \leq \frac{(n^2+1)^2}{n}.$$

Q.853 a) Show that for every positive integer n

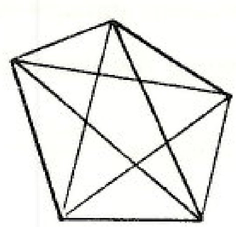
$$2\left(\sqrt{n+1} - \sqrt{n}\right) < \frac{1}{\sqrt{n}} < 2\left(\sqrt{n} - \sqrt{n-1}\right)$$

b) Find the largest integer less than

$$\sum_{k=1}^{10000} \frac{1}{\sqrt{k}}.$$

Q.854 The number 4 can be expressed as the sum of one or more positive integers in 8 ways: 4, 3+1, 2+2, 2+1+1, 1+3, 1+2+1, 1+1+2, 1+1+1+1. Note that the order of the summands is regarded as significant in this count; e.g. $2+1+1$ and $1+2+1$ and $1+1+2$ are all counted. Find a formula for the number of different ways in which an arbitrary positive integer n can be expressed as a sum of positive integers.

Q.855 The diagram shows a convex pentagon with all diagonals drawn.



They intersect in 5 points, which divide the diagonals into 15 line segments. The diagonals partition the interior of the pentagon into 11 regions. Given a convex n-gon with all diagonals drawn, no three of which are concurrent, find

(i) how many points of intersection are there;

(ii) into how many segments are the diagonals divided;

(iii) the number of regions into which the n-gon is partitioned by the diagonals.

Q.856 The senior form has three classes, all with twenty students. Each student is acquainted with forty other seniors. Prove that there is at least one set of three mutual acquaintances, one of whom is in each class.

Q.857 $a, b, c, \cdots, k$ is any set of n positive numbers.
Let $S = a + b + \cdots + k$ and $T = \frac{1}{a} + \frac{1}{b} + \cdots + \frac{1}{k}$.
Prove that $ST \geq n^2$.

Q.858 Let $a_1 = \sqrt{2}$; $a_2 = \sqrt{2}^{\sqrt{2}}$; $a_3 = \sqrt{2}^{\sqrt{2}^{\sqrt{2}}}$; $\cdots$; $a_n = \sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot^{\sqrt{2}}}}}}$
where there are n $\sqrt{2}$'s in the tower. Show that the list of numbers $a_1, a_2, \cdots, a_n, \cdots$ increases steadily, but that no matter how large n is, $a_n < 3$. Can you determine approximately how large a_n is when n is very big?

Q.859 In $\triangle ABC$, $AB = AC$. The bisector of $A\hat{B}C$ meets AC at D. If $BD + AD = BC$ find the angles of the triangle.

Q.860 Two congruent circles intersect at AB. Point P lies on one circle, and Q on the other. M, N are the feet of the perpendiculars from A to the lines BP and BQ respectively. Prove that the mid points of AB, PQ and MN are collinear.

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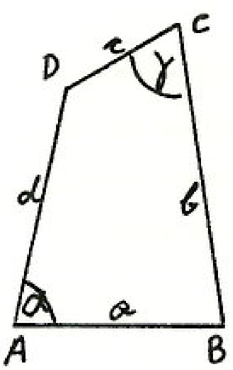

D. Bennewitz from Koblenz, Germany, has communicated (amongst other things) shorter workings of Q835 and Q838 based on the theorem:

$$(\text{Area } ABCD)^2 = (s-a)(s-b)(s-c)(s-d) - abcd\cos^2\frac{\alpha+\gamma}{2}$$

where $s = \dfrac{a+b+c+d}{2}$. Can you prove this theorem?