SOLUTIONS TO SCHOOL MATHEMATICS COMPETITION

JUNIOR

1. i) Find all positive numbers x which are such that $\left(1+\frac{1}{nx}\right)^{-1} > 1 - \frac{1}{n}$ for every positive integer n.

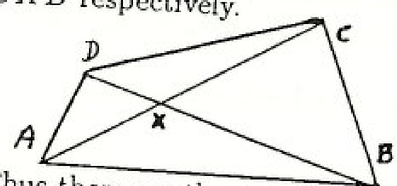
ii) The diagonals of a convex quadrilateral divide it into four triangles whose areas are 1,2,3 and a units (in some order). Find all the possible values of a.

Ans.i)

$$\left(1+\frac{1}{nx}\right)^{-1} > 1-\frac{1}{n} \Leftrightarrow \frac{nx}{1+nx} > \frac{n-1}{n}$$
$$\Leftrightarrow n^2x > n^2x + n - nx - 1$$
$$\Leftrightarrow x > \frac{n-1}{n} \Leftrightarrow x > 1 - \frac{1}{n}$$

By selecting a sufficiently large positive integer n, $1-\frac{1}{n}$ can be made arbitrarily close to 1. Hence x cannot be less than 1, but any value greater than or equal to 1 satisfies the condition.
Answer: $x \geq 1$.

ii) In the figure, let u, v, w, a be the areas of the triangles AXD, CXD, AXB, and CXB respectively.



Note that $u/v = AX/CX$ (since the areas are proportional to the base lengths if the heights are the same). Similarly $w/a = AX/CX$.
$\therefore a = vw/u$.

Thus there are three possible values of a when u, v, w are 1, 2 and 3 in some order viz.

$$a = \frac{1\times 2}{3} = \frac{1}{3};\ a = \frac{1\times 3}{2} = 1\frac{1}{2};\ \text{and } a = \frac{2\times 3}{1} = 6.$$

2. A game is played with 100 piles of pebbles, containing initially $1, 2, 3, \cdots, 99, 100$ pebbles. A move consists in choosing some of the piles and removing the same number of pebbles from each of them. For example one possible first move would be to remove 10 pebbles from each pile containing ten or more, thus leaving piles of sizes $1, 2, 3, \cdots 8, 9, 0, 1, 2, \cdots, 90$. The object of the game is to remove all the pebbles from the board in the smallest possible number of moves. How many moves are needed? Give a proof.

Ans. There are many different strategies which remove all the pebbles in seven moves. For example:-

1st move:- Take 50 pebbles from piles with more than 50; leaving

$$1, 2, \cdots, 49, 50; 1, 2, \cdots, 49, 50.$$

2nd move:- Take 25 pebbles from piles with more than 25; leaving

$$1, 2, \cdots, 25; 1, 2, \cdots, 25; 1, 2, \cdots, 25; 1, 2, \cdots, 25.$$

3rd move:- Take 13 pebbles from piles with more that 12; leaving 4 empty piles, and 8 piles with each of $1, 2, \cdots, 12$ pebbles.
4th move:- Take 6 pebbles from piles with more than 6; leaving 16 piles with each of $1, 2, \cdots 6$ pebbles.
5th move:- Take 3 pebbles from piles with more than 3; there are now 4 empty piles, 32 piles of size 1, 32 of size 2, and 32 of size 3.
6th move:- Take 2 pebbles from piles with two or more; there are now 36 empty piles, and 64 with one pebble.
7th move:- Take the last pebble from the 64 piles not yet empty.
We must prove that the pebbles cannot be removed in 6 moves. Suppose after the n th move, we can find k_n piles of the same size, but not $(k_n + 1)$ equal piles. For example $k_0 = 1$ since at the start no two piles have the same number of pebbles. For the sequence of moves given above, we have $k_2 = 4$, since after the second move we can find 4 equal piles (in many different ways), but not more than 4.
We claim that, for **any** sequence of moves $k_{n+1} \leq 2k_n$. This is clear, since the k_{n+1} equal piles after the $(n + 1)$th move can be placed into two groups; those which have been left unchanged by the $(n + 1)$th move, and those which have been reduced at the $(n + 1)$th move. Neither of these groups can have contained more than k_n piles.

Since $k_0 = 1$ it follows that $k_1 \leq 2$, $k_2 \leq 2^2$; $k_3 \leq 2^3$; and so on. Hence $k_6 \leq 2^6 = 64$. Hence there cannot be more than 64 empty piles after 6 moves. (In fact, taking the argument a step further, let E_n denote the number of empty piles after the n^{th} move, so that $E_0 = 0$, $E_{n+1} \leq E_n + k_n \leq E_n + 2^n$.

$$\therefore E_1 \leq 0 + 1, \; E_2 \leq 0 + 1 + 2; \; E_3 \leq 0 + 1 + 2 + 2^2; \text{ etc.}$$

Hence $E_6 \leq 1 + 2 + 2^2 + \cdots + 2^5 = 63$.)

3. Given the numbers A, B and C we are to solve for x, y and z the simultaneous equations

$$\begin{cases} xy + x + y = A \\ xz + x + z = B \\ yz + y + z = C. \end{cases}$$

i) Find all the solutions when $A = -7, B = 14$ and $C = -11$.

ii) Find a set of values for A, B and C which is such that the equations have no solution.

Ans. The equations can be rewritten

$$(x + 1)(y + 1) = A + 1$$
$$(x + 1)(z + 1) = B + 1$$
$$(y + 1)(z + 1) = C + 1.$$

Multiplying yields $[(x + 1)(y + 1)(z + 1)]^2 = (A + 1)(B + 1)(C + 1)$.

i) If $A = -7, B = 14, C = -11, (A+1)(B+1)(C+1) = 900 = 30^2$.

$$\therefore (x+1)(y+1)(z+1) = \pm 30.$$

$$x+1 = \frac{\pm 30}{C+1} = \pm 3; x = -4, \text{ or } +2$$
$$y+1 = \frac{\pm 30}{B+1} = \pm 2; y = 1, \text{ or } -3$$
$$z+1 = \frac{\pm 30}{A+1} = \pm 5; z = -6, \text{ or } +4.$$

There are two solutions:= $(x, y, z) = (-4, 1, -6)$; or $= (2, -3, 4)$.

ii) Since perfect squares are never negative, for solutions to exist $(A+1)(B+1)(C+1) = [(x+1)(y+1)(z+1)]^2$ must not be negative. (Also, if it is zero, at least one of $x+1, y+1, z+1$ must vanish, so at least two of $A+1, B+1, C+1$ must be zero).

For example there is no (real) solution if $(A, B, C) = (-2, 0, 0)$ or if $(A, B, C) = (-1, 0, 0)$.

4. i) How many positive integers x satisfy the equation

$$\left[\frac{x}{10}\right] = \left[\frac{x}{11}\right] + 1.$$

(Here $[x]$ denotes the largest integer less than or equal to x).

ii) Given a positive integer n, how many positive integers x satisfy

$$\left[\frac{x}{n}\right] = \left[\frac{x}{n+1}\right] + 1.$$

Ans. i) Let q be the quotient and r the remainder when x is divided by 11. i.e. $x = 11q + r$ where $0 \le r < 11$. Then $\frac{x}{11} = q + \frac{r}{11}$ so that $\left[\frac{x}{11}\right] = q$.
Now $\frac{x}{10} = q + \frac{q+r}{10}$ so that $\left[\frac{x}{10}\right] = q+1$ if and only if $1 \le \frac{q+r}{10} < 2$; i.e. $10 \le q + r < 20$.
For each of the eleven possible values of r (viz. $0, 1, 2, \cdots, 10$) there are 10 different integers q having $10 - r \le q < 20 - r$. Hence there are 110 different pairs (q, r), each giving rise to a different value of x. (e.g. $q = 3, r = 9$ corresponds to $x = 11q + r = 42$).
Hence there are 110 different integers x satisfying the equation.

ii) Similarly, let $x = (n+1)Q + R$ where $0 \le R < n+1$.
Then $\left[\frac{x}{n+1}\right] = Q$, and $\frac{x}{n} = Q + \frac{Q+R}{n}$. $\therefore \left[\frac{x}{n}\right] = \left[\frac{x}{n+1}\right] + 1$ if and only if $1 \le \frac{Q+R}{n} < 2$; i.e. $n - R \le Q < 2n - R$. For each of the $(n+1)$ different

i) If $A = -7, B = 14, C = -11, (A+1)(B+1)(C+1) = 900 = 30^2$.

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There are two solutions:$= (x, y, z) = (-4, 1, -6)$; or $= (2, -3, 5)$.

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possible values of R there are n (non-negative) integers Q in this range. These $n\times(n+1)$ different pairs (Q,R) correspond to $n(n+1)$ different values of x which satisfy the equation.

5. A list of positive integers $x_1, x_2, \cdots, x_n, \cdots$ satisfies the condition

$$x_n = x_{n-1} + [\sqrt{x_{n-1}}] \qquad \text{for all } n > 1.$$

(Here $[x]$ denotes the largest integer less than or equal to x.)
For example, if $x_1 = 19$, the list commences

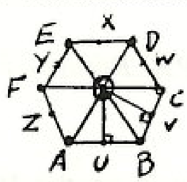
$$19, 23, 27, 32, 37, 43, 49, \cdots.$$

Prove that for any x_1 the list contains perfect squares.

Ans. See Senior 3 for answer.

6. Sixty seven points lie inside a regular hexagon with side length 2 centimetres. Prove that a circular coin of radius 1 centimetre can be placed to cover at least twelve of the points.

Ans. Join the centre 0 of the hexagon to the mid points U, V, W, X, Y, Z of the sides.



Since ΔOAB is equilateral, $OU \perp AB$. Therefore a circle on diameter OB passes through U (and V). Let $x_1, x_2, \cdots, x_6$ be the numbers of points lying inside or on the figures $OUBV$, $OVCW$, $\cdots$ respectively.

Then $x_1 + x_2 + \cdots + x_6 \geq 67$. (The equality sign applies if none of the given points lies on UX, VY or WZ). Hence the largest of $x_1, \cdots, x_6$ is at least 12. For definiteness let us suppose that the figure has been labelled so that $x_1 \geq 12$. Place the coin so that a diameter coincides with OB. Then it covers 12 of the points unless one of the points is at O (and $x_1 = 12$). But then since the 67 points are all inside the hexagon, none is at U, B or V and the centre of the coin can be shifted by a minute amount towards O to cover this point without uncovering any of the other eleven.

SENIOR

1. The set S consists of all positive integers which are factors of at least one of

$$1992,\ 6^{10},\ 18^{8}.$$

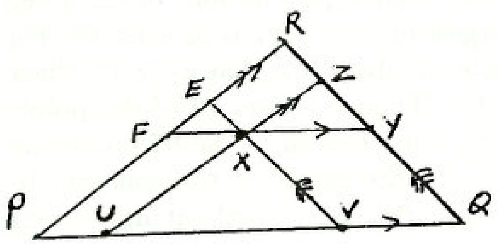
How many numbers are in S.

Ans. The divisors of $18^8 (= 2^8 3^{16})$ are the numbers $2^a 3^b$ where $a = 0, 1, 2, \cdots, 8$, and $b = 0, 1, 2, \cdots, 16$. There are 9×17 ways to choose a and b, giving 153 different elements of S.
In addition $2^e 3^f$ is a factor of $6^{10} (= 2^{10} 3^{10})$ not already counted, if $e = 9$ or 10 and $f = 0, 1, 2, \cdots, 10$. This gives another $2 \times 11 = 22$ elements of S. Since $1992 = 83 \times 2^3 \times 3$, its only factors not already counted are $83 \times 2^h \times 3^k$ where $h = 0, 1, 2$, or 3, and $k = 0$ or 1. This gives another $4 \times 2 = 8$ elements of S. Hence altogether S has $153 + 22 + 8 = 183$ elements.

2. Through a point inside a triangle of area A are drawn three lines parallel to the sides of the triangle. These lines partition the interior of the triangle into three parallelograms and three triangles. If the triangles have areas A_1, A_2 and A_3, prove that

$$\sqrt{A_1} + \sqrt{A_2} + \sqrt{A_3} = \sqrt{A}$$

Ans. If A denotes the area of a triangle, $\triangle PQR$, $A = \frac{1}{2} PR.\ PQ \sin \angle P = kPQ^2$ where $k = \frac{1}{2}\frac{PR}{PQ} \sin \angle P$.



Likewise, any triangle similar to $\triangle PQR$ has its area given by $k\ell^2$, where ℓ is the length of the side corresponding to PQ, since $\angle P$, and the ratio PR/PQ are both unchanged by similarity.
In the figure, the triangles $\triangle PQR$, $\triangle UVX$, $\triangle XYZ$ and $\triangle FXE$ are equiangular because of the parallel lines, and therefore similar. If their areas are A, A_1, A_2, A_3, respectively, then from the above

$$\sqrt{A} = \sqrt{k}PQ,\ \sqrt{A_1} = \sqrt{k}UV,\ \sqrt{A_2} = \sqrt{k}XY, \text{ and } \sqrt{A_3} = \sqrt{k}FX.$$

$$\begin{aligned}\therefore \sqrt{A_1} + \sqrt{A_2} + \sqrt{A_3} &= \sqrt{k}(UV + XY + FX) \\ &= \sqrt{k}(UV + VQ + PU)\end{aligned}$$

(since opposite sides of parallellograms are equal)

$$= \sqrt{k}PQ = \sqrt{A}.$$

3. A list of positive integers

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satisfies the condition

$$x_n = x_{n-1} + [\sqrt{x_{n-1}}] \qquad \text{for all } n > 1.$$

(Here $[x]$ denotes the largest integer less than or equal to x.)
For example, if $x_1 = 19$, the list commences

$$19, 23, 27, 32, 37, 43, 49, \cdots.$$

i) Prove that for any x_1 the list contains perfect squares.

ii) Find a number x_1 which is such that the first perfect square in the list is x_{20}.

Ans. i) It will be sufficient to show that at least one perfect square appears, since then there will be another square in the continuation of the list starting at the next term. Note that if x is a term in the list such that $k^2 \leq x < (k+1)^2$, then $[\sqrt{x}] = k$, and the following term is $x + k$.

Now suppose $(m-1)^2 < x_1 < m^2$ (where $m \geq 2$). Let x_j be the first member of the list such that $x_j \geq m^2$. If $x_j = m^2$ we are finished, since we have found a perfect square. Otherwise $x_j = m^2 + \delta$ where $1 \leq \delta < m - 1$ (since x_j exceeds x_{j-1} by only $m-1$). Now $x_{j+1} = x_j + m < m^2 + 2m - 1 < (m+1)^2$ and $x_{j+2} = x_{j+1} + m = x_j + 2m = m^2 + 2m + \delta = (m+1)^2 + (\delta - 1)$. This analysis shows that for any m, there are always two terms of the list between m^2 and $(m+1)^2$ and if the smaller of them x_j exceeds m^2 by δ, the term x_{j+2} exceeds $(m+1)^2$ by $\delta - 1$. If $\delta - 1 > 0$, x_{j+4} exceeds $(m+2)^2$ by $(\delta - 2)$, and so on. It is clear that after δ pairs we reach a term in the list ($x_{j+2\delta}$ in fact) which exceeds a perfect square by 0. i.e.

$$x_{j+2\delta} = (m + \delta)^2.$$

ii) Taking $j = 2, \delta = 9$ in the above,

$$x_2 = m^2 + 9 \quad (\text{where } 9 < m - 1)$$

and $x_{20} = (m+9)^2$.
The smallest permissible value of m is 11. Taking this

$$\therefore x_2 = 130 = x_1 + 10 \quad (\text{since } 10^2 < x_1 < 11^2)$$

$\therefore x_1 = 120$ is one possible value. (In fact the smallest). Other possible values of x_1 are $x_2 - (m-1)$

$$x_1 = (m^2 + 9) - (m - 1) = m^2 - m + 10 \text{ (for any } m \geq 11)$$

4. At a shooting gallery there are n targets in a row. A customer knocked over all the targets in n shots. At no stage was any target left standing after its neighbours to

the left and the right had both been knocked over. In how many different orders could the targets have been hit?

Ans. 1. Let N_n denote the required answer. Obviously $N_1 = 1; N_2 = 2$. Also $N_3 = 4$, since the targets ABC can be hit in the orders A, B, C; or B, A, C; or B, C, A; or C, B, A.

The last target to be hit must be at one end of the row. If it is at the left hand end, there are N_{n-1} orders in which the other $(n-1)$ targets can be knocked over. Similarly, there are another N_{n-1} ways to hit the targets if we end at the right hand target. Hence for all $n > 1$ $N_n = N_{n-1} + N_{n-1} = 2 \times N_{n-1}$.

Hence, since $N_1 = 1$, we obtain $N_n = 2^{n-1}$ for all $n \geq 1$.

2. Alternatively, suppose the first target to be hit is the kth target from the left hand end. There remain $k-1$ targets to its left and $n-k$ to its right. The next target to be hit must be either target $k+1$, or target $k-1$, which we denote by R and L respectively. In fact, the order of hitting the remaining $n-1$ targets can be described by a sequence $X_1 X_2 \cdots X_{n-1}$ where $k-1$ of the X's stand for L, and the others stand for R. For example, the sequence $\underset{(k-1)}{L\,L \cdots L}\ \underset{(n-k)}{R\,R \cdots R}$ would correspond to the order of hitting the targets by working along to the left hand end of the row first, and then working to the right hand end. Since there are ${}^{n-1}C_{k-1}$ ways to choose $(k-1)$ of the $n-1$ places for the L's, there are exactly that many different orders for hitting the targets starting with the kth. Adding for $k = 1, 2, \cdots n$,

$$\begin{aligned} N_n &= {}^{n-1}C_{1-1} + {}^{n-1}C_{2-1} + {}^{n-1}C_{3-1} + \cdots + {}^{n-1}C_{n-1} \\ &= (1+1)^{n-1} \text{ (by the binomial theorem)} \\ &= 2^{n-1} \end{aligned}$$

5. S is a set of non-zero numbers, no two equal. If two members of S are selected at random, their product is equally likely to be positive as negative. The probability that they are both positive is the same as the probability that if one member of S is chosen at random it is negative.
How many numbers are in S.

Ans. Suppose p of the numbers are positive, and n negative. There are $\frac{p(p-1)}{2}$ ways to choose 2 positive numbers, $\frac{n(n-1)}{2}$ ways to choose 2 negative numbers, and $p \times n$ ways to choose one of each. Since the product of two numbers is equally likely to be positive as negative, we obtain

$$\frac{p(p-1)}{2} + \frac{n(n-1)}{2} = p \times n. \qquad (1)$$

The probability that both numbers chosen is positive is $\frac{p(p-1)}{2} / \frac{(p+n)(p+n-1)}{2}$, and this must be equal to $\frac{n}{p+n}$.

$$\therefore \frac{p(p-1)}{(p+n)(p+n-1)} = \frac{n}{p+n} \qquad (2)$$

From (1) $\quad p(p-1) = n(2p-n+1)$

and from (2) $\quad p(p-1) = n(p+n-1) \qquad (3)$

Hence $p+n-1 = 2p-n+1$, giving $p = 2n-2$.

Substitute in (3). $(2n-2)(2n-3) = n(3n-3)$

$$\therefore n = 1 \text{ or } n = 6.$$

$$p = 2n-2 = 0 \text{ or } 10.$$

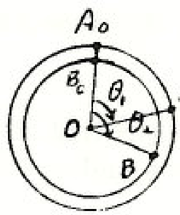
The wording of the question does not make sense if S contains only 1 number. Hence $n = 6, p = 10$ is the only solution, S containing 16 numbers.

6. Two marathon runners are training on a circular track marked with lanes. They both run clockwise at a steady speed of 5 metres/sec, one in an outside lane which has a radius of 100 metres, the other in a concentric lane with a radius of 96 metres. When the distance between them is changing most rapidly,

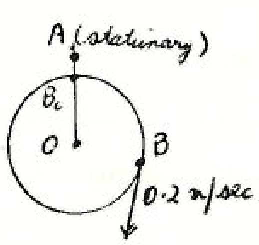
i) at what rate is it changing?

ii) how far apart are the runners?

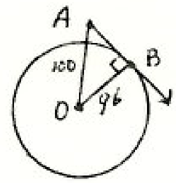
Ans. Suppose we measure time, t, from the moment when the two runners are closest together, their positions obviously collinear with the centre of the circles, O. Let θ_1 (radians) be the angle at O travelled by the outside runner A after t seconds, and θ_2 the corresponding angle for the runner B on the inside track. Then $100\theta_1 = 5t$ and $96\theta_2 = 5t$ so $\frac{\theta_2 - \theta_1}{t} = 5(\frac{1}{96} - \frac{1}{100}) = \frac{1}{5 \times 96}$ (radians/sec).



Hence the angle subtended at O by the line segment AB is changing at a constant rate of $\frac{1}{5 \times 96}$ radians/sec. Therefore their distance apart after t seconds is exactly the same as if A was stationary at his position at $t = 0$ while B travelled along the inside lane at a constant speed of $96 \times (\frac{1}{5 \times 96})$ meters/sec. (i.e. at .2 m/sec.)



Now it is obvious in this situation that the distance between A and B changes most rapidly when B is moving either directly away from (or else directly towards) A's position.



i.e. when the line AB is tangential to the smaller circle. By Pythagoras theorem, the distance between the runners is given by $\sqrt{100^2 - 96^2} = 28$ metres, and it is increasing (or decreasing) at the rate of 0.2 m/sec.

7. The sum of the lengths of the twelve edges of a rectangular box is 24 metres, and the sum of the areas of the six faces is 18 square metres. What is the largest possible volume of the box?

Ans. Let the dimensions of the box be x, y, z metres. We are given $4(x + y + z) = 24$ and $2(xy + xz + yz) = 18$. Hence we want to find the largest value of $V = xyz$ where x, y, z are positive numbers satisfying

$$x + y + z = 6 \quad (1)$$

and

$$xy + xz + yz = 9 \quad (2)$$

From (1) and (2)

$$\begin{aligned} yz &= 9 - x(y + z) \\ &= 9 - x(6 - x) \\ &= (x - 3)^2. \end{aligned}$$

Remembering that for positive numbers y, z

$$yz \leq \left(\frac{y + z}{2}\right)^2$$

we have $(x - 3)^2 \leq \frac{(6 - x)^2}{4}$ which simplifies to $x \leq 4$.

So we want the largest value of $V = x(yz) = x(x-3)^2$ when x ranges between 0 and 4.

EITHER

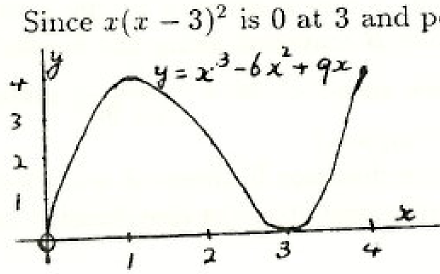
Since $(x(x - 3)^2 = 4 - (4 - x)(x - 1)^2$ it is clear that the largest value achievable by V (for $x \leq 4$) is 4, obtained when $x = 4$ and $y = z = 1$ (and also when $x = 1,\ y = 4,\ z = 1;\ x = 1,\ y = 1,\ z = 4$).

OR

Use calculus to clarify the graph of $V = x(x - 3)^2 = x^3 - 6x^2 + 9x$ for $0 \leq x \leq 4$.

$$\frac{dV}{dx} = 3x^2 - 12x + 9 = 0 \text{ when } x = 1 \text{ or } x = 3.$$

Since $x(x - 3)^2$ is 0 at 3 and positive elsewhere in $[0, 4]$, $x = 3$ is a minimum.



Since $\frac{d^2V}{dx^2} = 6x - 12$ is negative at $x = 1$, $x = 1$ is a maximum.

Therefore the graph of V in $0 \leq x \leq 4$ looks like the sketch. Since $V(1) = 1 \times (-2)^2 = 4$ and $V(4) = 4 \times 1^2 = 4$ the largest value of V is 4.