## PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

**Q.861** Each of the numbers in a list

$$x_1, x_2, x_3, \cdots, x_n, \cdots$$

is a positive integer written as usual in decimal notation.

For every $n > 1$

$$x_n = x_{n-1} + y_{n-1}$$

where $y_{n-1}$ is the number obtained from $x_{n-1}$ by writing down the digits in reverse order. For example, if $x_n = 100$, then $x_{n+1} = 100 + 001 = 101$, and $x_{n+2} = 101 + 101 = 202$.

Prove that regardless of the value of $x_1$ from some stage on all the numbers in the list are exactly divisible by 11.

**Q.862** **(i)** Sketch the graph of the function $f(x) = \frac{\ell n x}{x} (x > 0)$.

(Here $\ell n x = \log_e x$; $\frac{d}{dx}(\ell n x) = \frac{1}{x}$).

**(ii)** Show that the only solution of $a^b = b^a$ where $a$ and $b$ are positive integers and $a < b$, is given by $a = 2, b = 4$.

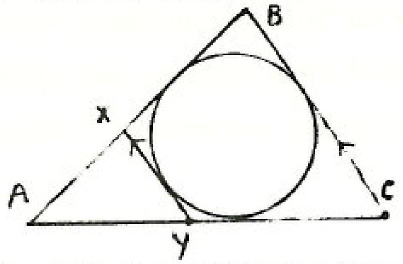
**Q.863** **(i)** Sketch the graph of the function $g(x) = x \ell n x (x > 0)$.

**(ii)** For $c > 0$ let $N(c)$ denote the number of solutions of $x^x = c$, $(x > 0)$. Find $N(c)$.

**(iii)** Find all solutions of $x^x = c$ when $c = \frac{\sqrt{2}}{2}$, when $c = \frac{4\sqrt[3]{36}}{9}$, and when $c = \frac{1}{2}$.

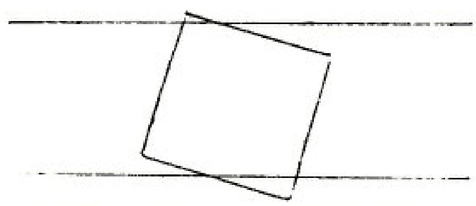
**Q.864** In a club with 36 members any two members are either friends or enemies, and each member has exactly 13 enemies. In how many different ways can one select three members so that they are either all friends or all enemies.

**Q.865** For any triangle $\triangle ABC$ let $X, Y$ be points in the sides $AB, AC$ respectively such that $XY \| BC$ and $XY$ is tangential to the inscribed circle of the triangle. (See figure).



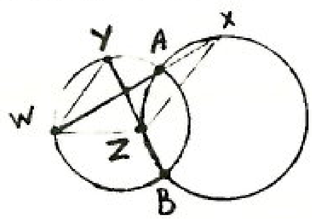

Prove that the length $XY$ cannot exceed $\frac{1}{8}$th of the perimeter of $\triangle ABC$. Is equality possible?

**Q.866** A 1 metre square masonry slab which formed part of a 1 metre wide path has become displaced as shown in the figure.



A workman repairing the path saws off the two triangular pieces which project beyond the sides of the path. Find the sum of the perimeters of the two triangles.

**Q.867** Two circles intersect in $A$ and $B$. $P$ is a point inside one of the circles.




$AP$ (produced) cuts the first circle in $W$, the second in $X$. $BP$ (produced) cuts the first circle in $Y$, the second in $Z$. If $WYXZ$ is a rhombus, prove that the two circles are of equal radius.

**Q.868** Twelve sentry posts are situated (not necessarily equally spaced) on the circular wall of a citadel. At 12 noon, a sentry leaves each post in one direction or the other, marching at a speed which would make a complete circuit in exactly one hour. If two sentries meet, they both about turn and continue marching at the same speed in opposite directions.

Prove that at midnight each sentry will be exactly at his own starting post.

In fact prove that this must already be the case at 6pm.

**Q.869** Let $c_n$ be the $n$th term of the sequence defined by $c_1 = 1,\ c_2 = -1,$

$c_n = -c_{n-1} - 2c_{n-2}$ for $n \geq 3$.

Prove that $2^{n+1} - 7c_{n-1}^2$ is a perfect square for every integer $n \geq 2$.

Continued p.32.